Abstracts

Hyperbolic Differential Equations, Calculus of Variations and Applications

July 17-19, 2024,
Heraklion, Greece

Abstracts

Hyperbolic Differential Equations, Calculus of Variations and Applications,
July 17-19, 2024,
Heraklion, Greece

Hyperbolic Differential Equations, Calculus of Variations and Applications

Preface

This volume contains the abstracts of the talks at the Hyperbolic Differential Equations, Calculus of Variations and Applications workshop held at Heraklion, Greece, on July 17-19, 2024. The workshop has been organized under the Institute of Applied and Computational Mathematics (IACM) at the Foundation for Research and Technology Hellas (FORTH).

The aim of this workshop has been to promote applied mathematics in Greece.

We would like to thank all contributors for submitting their abstracts and presenting their work at the workshop.

Charalambos Makridakis
Athanasios Tzavaras

Organizing Committee

Charalambos Makridakis	FORTH	c.g.makridakis@iacm.forth.gr
Athanasios Tzavaras	KAUST	athanasios.tzavaras@kaust.edu.sa

Zero Relaxation Limit in BV

Constantine Dafermos
Brown University

The lecture will survey progress and open problems in a research program aiming at the construction of BV solutions to hyperbolic systems of balance laws with a stiff source, modeling relaxation, and the passage to the zero relaxation limit.

Multi-phase minimizers for the Allen-Cahn system on the plane

Nicholas Alikakos
University of Athens

In this talk we investigate multi-phase minimizers for the Allen-Cahn system on the plane. Our emphasis is on distinct surface tension coefficients. The proofs do not rely on symmetry. Coexistence of an arbitrary number of phases is related to the existence of the relevant minimizing cones for the minimal partition problem. For example, the orthogonal cross with four phases is minimizing for a certain class of surface tension coefficients. We focus on two examples: the entire solution for the triple junction, and a four-phase minimizer with three-phase Dirichlet data (the triangle). The results presented in the talk are based on joint work with Zhiyuan Geng (Triple Junction), and with Dimitrios Gazoulis (The Triangle).

Why 99.8% of the time there are no Rogue Waves in the North Atlantic (The answer is Landau damping)

Agissilaos Athanassoulis
University of Dundee

The Alber equation was proposed by I. E. Alber as a statistical model for nonlinear stochastic ocean waves in 1978.
Since then, it has been considered a key ingredient for the understanding of Rogue Waves in ocean engineering, and features in field manuals and ship-registers' position papers. However, numerous open questions remained until recently, limiting its quantitative use. In this talk we will present the stability theory for this equation, developed in collaboration with Themis Sapsis (MIT), Mariya Ptashnyk (Heriot-Watt) and Gerassimos Athanassoulis (NTUA). This includes the state of the art for Landau damping (which describes the usual stable ocean) and the bifurcation into instability (which seems to be responsible for Rogue Waves).

Crucially, Landau damping is possible even under non-mean-zero perturbations. This is significant for ocean waves, and there is physical reason to expect that it would extend to the fully nonlinear case, unlike the classical Vlasov equation, where the respective constraint is that any inhomogeneity must not carry net charge. If time allows, the links with other open questions will be explored (e.g. global existence for the focusing infinite system of fermions; derivation of the so-called wave kinetic equation; and real-world oceanic rogue waves).

Islands of stability and periodic waves in the Anti-de Sitter spacetime

Athanasios Chatzikaleas
Universitat Münster, Germany

In 2006, Dafermos-Holzegel conjectured that the Anti-de Sitter spacetime is an unstable solution to the Einstein equations with negative cosmological constant for general initial data under reflective boundary conditions. This conjecture is now rigorously proved by a fascinating method developed by Moschidis in 2023. Moreover, Rostworowski-Maliborski enhanced the conjecture by providing numerical evidence that indicate the existence of "special" initial data leading to time-periodic solutions for the Einstein-Klein-Gordon system which are in fact stable. Motivated by their numerical arguments, we present joint works with Jacques Smulevici (Sorbonne Universite) concerning the construction of families of time-periodic solutions to non-linear massive Klein-Gordon equations in fixed Anti-de Sitter as well as their orbital stability over exponentially long times.

On hyperbolic balance laws with application to self-organized systems of Euler-type

Cleopatra Christoforou
University of Cyprus

Mathematical models introduced to capture the emergent behavior of self-organized systems have brought new mathematical challenges and most studies on flocking models investigate so far smooth solutions. In this talk, we will first have an overview of the theory of entropy weak solutions to hyperbolic balance laws and then describe a hydrodynamic model of flocking-type with an all-to-all interaction kernel. The system with linear pressure, that is under consideration, is inspired by a hydrodynamic model obtained as a limit in a kinetic equation of Cucker–Smale type with stochastic forcing. We will present recent results on this topic in the context of weak solutions; more precisely, we will discuss how the theory of balance laws is further developed to establish the global existence of entropy weak solutions to Euler-type flocking systems with arbitrary initial data that are confined within a bounded region and also capture unconditional time-asymptotic flocking behavior for weak solutions.

Global stability of cosmological fluids with extreme tilt

Grigorios Fournodavlos
University of Crete

In cosmology, the equation of state of a perfect fluid is considered to be $p = c_s^2 \rho$, where c_s is the speed of sound. The simplest solution to the Einstein-Euler system, known as FLRW, representing a cosmological fluid, was discovered by Friedmann already in 1922. There is an extensive literature in physics concerning the dynamics of cosmological fluids. However, rigorous mathematical works proving the stability of homogeneous backgrounds are so far restricted to small sound speeds, up to the radiation threshold. Interesting bifurcation phenomena and instabilities are predicted for larger sound speeds. I will discuss joint work with E. Marshall and T. A. Oliynyk proving the global stability of homogeneous solutions with so-called extreme tilt, whose fluid vector field becomes asymptotically null, beyond the radiation case.

Grain boundary compatibility for martensitic phase transformations

John Ball* and Myrto Galanopoulou#
*Department of Mathematics, Heriot-Watt University, Edinburgh
#Department of Mathematics, University of Sussex, Brighton

We study compatibility across grain boundaries of polycrystals undergoing martensitic phase transformations. For the simplest case of a planar grain boundary we consider when it is possible to have a constant deformation gradient of minimum energy in each grain, giving a necessary and sufficient condition in terms of the energy well structure and relative rotation of the crystal axes. In general, this condition is not satisfied, so that microstructure is required to achieve compatibility. We discuss possible generalizations when the constant deformation gradients are replaced by laminates of given orders.
Further, we discuss the set S of macroscopic deformation gradients allowing for underlying zero-energy microstructures independent of grain geometry and grain rotations, showing that for cubic austenite this is a nonempty relatively open set in the set of 3X3 matrices with determinant equal to that of the transformation strain. For a cubic to tetragonal transformation an upper bound for S is given in terms of the set D of positive diagonal matrices in the quasiconvex hull of the energy wells. The set D has been determined by Peigney (J. Mech. Phys. Solids 61 (2013) 1489–1510), and we give a simpler proof of his result.

Towards new field theories for fracture, plasticity and cancer growth, through nonconvex calculus of variations, bifurcation theory for PDE, and modelling in nonlinear elasticity.

Phoebus Rosakis
University of Crete

Traditional approaches to challenging phenomena in solid mechanics have long been shackled by linear elastic thinking (stemming from the venerable, mathematically well understood, but approximate theory of Linear(ized) Elasticity) even when venturing into the regime of nonlinearity. This means that fracture, dislocations, plasticity and even phase transitions are typically described by imposing ad hoc conditions (for fracture, plastic flow or phase nucleation) upon linear or mildly nonlinear models that are pulled off the shelf in an equally ad hoc manner.

Nonlinear elasticity enjoys a great freedom that should be attractive to the modeler, but is largely unexploited because of fear of the unknown. This is the choice of the energy functional. I will present three examples, where multiscale modelling hopefully gives rise to a full field theory. Here "field theory" roughly means that the phenomenon described emerges spontaneously and evolves naturally, without extraneous ad hoc conditions, but because of the unusual nonlinearity of the energy functional (deduced by careful modeling that often involves multiple scales and symmetry). These phenomena are fibrosis in biological tissues, brittle fracture and dislocation/plasticity evolution in crystals (the latter due to L. Truskinovsky and coworkers). They share a common theme, inferred from subtle modelling. The energy functional exhibits nonconvexity (of very different types). This leads to instability that can be understood using various tools: conservations laws that change type, nonconvex calculus of variations, bifurcation theory for PDE, careful numerical analysis, and even, heaven forbid, neural networks.

Mean Curvature flow Singularities

Panagiota Daskalopoulos
Columbia University

< ABSTRACT >

Renormalized solutions for the Maxwell-Stefan system with an application to uniqueness of weak solutions

Stefanos Georgiadis
KAUST and TU Vienna

Cross-diffusion systems are strongly coupled parabolic systems describing phenomena in which multiple species diffuse and interact with one another, e.g. in fluid mechanics or population dynamics. Although many methods have been developed to study the existence of weak solutions to such systems, uniqueness is in general an open problem. To this degree, we study a particular cross-diffusion system, known as the Maxwell--Stefan system which describes diffusive phenomena in a multicomponent system of gases. We employ renormalized solutions and give conditions under which such solutions are unique. We, then, study the relation between weak and renormalized solutions, allowing us to identify conditions that guarantee uniqueness of weak solutions. The proof is based on an identity for the evolution of the symmetrized relative entropy. Using the method of doubling the variables we derive the identity for two renormalized solutions and use information on the spectrum of the Maxwell-Stefan matrix to estimate the symmetrized relative entropy and show uniqueness. This is a joint work with Hoyoun Kim and Athanasios Tzavaras.

Hypocoercivity-preserving Galerkin discretisations of kinetic equations

Emmanouil Georgoulis
Herriot Watt Univ and Natnl Techn. Univ of Athens

Numerous physical, chemical, biological, and social dynamic processes are characterised by convergence to long-time equilibria. These are often described as PDEs of kinetic type, whereby ``position" and ``velocity" are independent variables; well-known examples of such are Kolmogorov and Fokker-Planck equations. These may also arise when modelling multi-agent interacting processes of particles, individuals, etc. In many important cases the diffusion/dissipation required to arrive to such equilibria is explicitly present in some of the spatial directions only, that is there exist evolution PDEs with degenerate diffusion yet converging to equilibrium states as time goes to infinity. This, somewhat counter-intuitive at first, state of affairs suggests that decay to equilibrium is due to finer hidden structure, which allows for the transport terms to also "propagate dissipation" to the spatial directions in which no dissipation appears explicitly in the PDE model. This property has been studied extensively by Villani who coined the term “hypocoercivity" to describe it in his celebrated 2009 AMS Memoir.

I will present recent results on how to design and analyse hypocoercivity-preserving Galerkin discretisations, in an effort to port the concept of hypocoercivity in the design and analysis of numerical methods. To that end I plan to start by presenting provably hypocoercivity-preserving Galerkin (non-conforming) finite element methods for the simple model problem of Kolmogorov equation. I plan to conclude with some first results of Galerkin methods for the inhomogeneous Fokker-Plack equation on exponentially weighted function spaces. Some of the results are joint work with Zhaonan Dong (INRIA, Paris) and Philip Herbert (Sussex).

Higher order corrections to the approximation of the 2d dual semi-geostrophic equation by the Euler vorticity equation

Ioannis Giannoulis
University of Ioannina

The dual semi-geostrophic equation is a coupled transport/Monge-Ampere system, while the 2d Euler equation in vorticity form is a coupled transport/Poisson system, which possesses smooth solutions globally in time. G. Loeper showed in 2006 that at a macroscopic time scale, solutions of the former can be approximated by solutions of the latter over arbitrary large finite macroscopic times. We present recent results of ongoing work concerning the derivation and justification of equations describing the dynamics of higher-order corrections to the leading order approximation by the Euler vorticity equation, thus refining the result of Loeper.
(This is joint work with V. Kalivopoulos in the framework of a project financed by HFRI-ΕΛΙΔΕΚ.)

On the role of quasiconvexity in dynamics

Konstantinos Koumatos
University of Sussex

Quasiconvexity is the central notion of convexity for vectorial problems in the calculus of variations ensuring the lower semicontinuity of integral functionals with respect to the weak topology in Sobolev spaces, and also plays a crucial role in the so-called Weierstrass problem, i.e. the problem of determining necessary and sufficient conditions for a given map to be a strong local minimiser of an integral functional. However, its role in dynamics is less clear. Motivated by work of C. Dafermos in systems of conservation laws possessing an involution, as well as recent advances in elliptic regularity, we establish a Gårding-type inequality for a quantity known as the relative entropy when the entropy is quasiconvex, or more generally A-quasiconvex. This allows us to adapt the well-known relative entropy method to prove weak-strong uniqueness results in (thermo)elastodynamics, and more generally for systems of conservation laws with an involution, when the entropy is (A-) quasiconvex.
This is a series of joint works with M. Galanopoulou, S. Spirito, and A. Vikelis.

Initial-boundary value problems for nonlinear Schrödinger-type equations

Dionyssis Mantzavinos
Kansas University

We will discuss the Hadamard well-posedness (existence and uniqueness of solution; continuity of the data-to-solution map) of certain nonlinear dispersive partial differential equations of Schrödinger type in the context of initial-boundary value problems. Such problems arise naturally in applications where the spatial domain involves a boundary. In these cases, apart from the usual initial conditions that are present in initial value (Cauchy) problems, it is necessary to also prescribe appropriate (nonzero) boundary conditions. We will focus on how these boundary conditions affect the well-posedness theory, importantly in regard to the method and techniques that must be specifically developed for establishing such a theory in the initial-boundary value problem setting.

Sharp convergence rates for mean field control in the region of strong regularity

Nikiforos Mimikos
University of Chicago

This is joint work with P. Cardaliaguet, J. Jackson, and P. E. Souganidis.
We study the convergence problem for mean field control, also known as optimal control of McKean-Vlasov dynamics. We assume that the data is smooth but not convex, and thus the limiting value function U: [0, T] × P2(Rd) → R may not be differentiable. In this setting, the first and last named authors recently identified an open and dense set O on which the limiting value function is C1, and solves the relevant infinite-dimensional Hamilton-Jacobi equation in a classical sense. In the present paper, we use these regularity results (and some non-trivial extensions of them) to derive sharp rates of convergence. In particular, we show that the value functions for the N-particle control problems converge towards U with a rate of 1/N, uniformly on subsets of O which are compact in the p-Wasserstein space for some p > 2. A similar result is also established at the level of the optimal feedback controls. Importantly, the rate 1/N is the optimal rate in this setting even if U is smooth, and the optimal global rate of convergence is known to be slower than 1/N. Thus our results show that the optimal rate of convergence is faster inside of O than it is outside. As a consequence of the convergence of optimal feedbacks, we obtain a concentration inequality for optimal trajectories of the N-particle problem started from i.i.d. initial conditions.

The role of continuum and discrete energies in machine learning algorithms

Charalambos Makridakis
IACM-FORTH and University of Crete

< ABSTRACT >

On a model of relativistic strings

Manoussos Grillakis
University of Maryland

I will talk about a certain model of a relativistic string proposed by F. Lund and T. Regge. The model is the relativistic analog of an incompressible uid with vorticity concentrated on a curve. The extra ingrediend being the addition of what is called a Nambu action in the model.

On linearisation around singular Rayleigh-Jeans for the 4-waves kinetic equation

Angeliki Menegaki
Imperial College

We consider the 4-waves spatially homogeneous kinetic equation arising in weak wave turbulence theory. In this talk I will present some new results on the existence and behaviour of solutions around different Rayleigh-Jeans (RJ) thermodynamic equilibria. In particular, I will discuss existence of global solutions and stability of *non-singular* RJ, under the assumption of confined frequencies. Moreover, I will present a more recent work on linearisation around *singular* RJ, where instabilities are present. If time permits, I will briefly discuss the nonlinear problem for singular initial data, where we prove a condensation in finite time, a fact that explains the behaviour of the singular linearised problem. The latter is a joint work with Miguel Escobedo (UPV/EHU).

Surface morphoelasticity

Katerina Nik
T.U. Delft

Growth is a fundamental process in biological systems and various technological applications, including epitaxial deposition and additive manufacturing. The interaction between growth and mechanics in deformable bodies leads to a wealth of very challenging mathematical questions. I will give a short overview of the key concepts of morphoelasticity, namely, the theory of elastic deformations in growing bodies. Unlike the classical case, the reference state of a growing body evolves over time, also in response to external stimuli and stress. I will discuss the case of surface accretion, which presents specific challenges. The emphasis will be on developing a variational framework where the existence of quasistatic morphoelastic evolution can be proved. This talk is based on a collaboration with Elisa Davoli (TU Wien), Ulisse Stefanelli (University of Vienna), and Giuseppe Tomassetti (Roma Tre University).

A generalization of Young measures for the Hydrodynamic limit of condensing Zero Range Processes

Marios Stamatakis
University of Ioannina

Zero-range processes are stochastic interacting particle systems with zero range interaction. For particular choices of their parameters they exhibit phase separation with the emergence of a condensate. Such zero-range processes are referred to as condensing and their hydrodynamic limit is not known. It is expected to be given by a degenerate non-linear diffusion equation where the diffusivity vanishes above a critical density ρc. We present a generalization of the notion of Young-measures which allows us to obtain a closed equation as the hydrodynamic limit of condensing zero-range processes. We focus on symmetric processes on the discrete torus and prove that the law of the empirical density of the zero-range process in terms of generalized Young-measures is concentrated on generalized Young measure-valued weak solutions $\pi = (\pi t)\ t \geq 0$ of the degenerate parabolic equation $\partial t \pi = \Delta\Phi(\pi)$, where $\Phi(\rho)$, $\rho \geq 0$, is the mean local jump rate of particles under the grand canonical equilibrium state of density $\min\{\rho, \rho c\}$.

Oscillations in hyperbolic-parabolic systems and the issue of homogenization

Athanasios Tzavaras
KAUST

We review several examples of oscillating solutions for hyperbolic-parabolic systems, including systems used in the dynamics of phase transitions as well as the compressible Navier-Stokes equations with non-monotone pressures.
We discuss the problem of obtaining effective equations for the description of oscillations.

Measure-valued solutions for non-associative finite plasticity

Andreas Vikelis
University of Vienna

The variational treatment of evolutionary nonassociative elasto-plasticity at finite strains remains unexplored. In this direction, following the concept of energetic solutions, we present an existence result for measure-valued solutions of the quasistatic evolution problem which are stable and balance the energy. In particular, we apply a modification of the standard timediscretization scheme, considering Young measures generated by piecewise constant interpolants of time-discrete solutions of a properly defined minimization problem. A key point in our analysis is the limit passage in the dissipation. The later calls for time-continuity properties of the stresses which are not expected in the quasistatic framework. To overcome this obstacle we introduce a regularization of the generalized stress in the definition of our energetic solutions.
Joint work with Ulisse Stefanelli.

Stability aspects of the Möbius group of $\mathbb{S}^{n-1}$

Konstantinos Zemas
University of Bonn

We will discuss quantitative stability aspects of the class of Möbius transformations of the sphere among maps in the critical Sobolev space (with respect to the dimension).

The special case of $\mathbb{S}^{n-1}$- and the more general case of $\mathbb{R}^n$-valued maps will be addressed. In the latter, more flexible setting, unlike similar in flavour results for maps defined on domains, not only a conformal deficit is necessary, but also a deficit measuring the distortion of $\mathbb{S}^{n-1}$ under the maps in consideration, which is introduced as an associated isoperimetric deficit.

The talk will be based on previous works in collaboration with Stephan Luckhaus, Jonas Hirsch, and more recent ones with Andre Guerra and Xavier Lamy.

Extremal and near extremal black holes

Mihalis Dafermos
Princeton University and University of Cambridge

Extremal (maximally rotating or maximally charged) and near-extremal black holes are of intense interest both for real astrophysics and in the context of fashionable speculations in high energy physics. In this talk, I will first introduce extremal black holes to a general pde audience, and then I will describe how a series of mathematical theorems proven over the past ten years (with reasonably short proofs!) have overturned many traditional expectations ---even supposed "laws"--- involving these objects, giving rise to a new conjectural picture of extremal gravitational collapse with both observational and theoretical significance.

Control of the Dyson equation and large deviations for the eigenvalues of large symmetric matrices

Panagiotis E. Souganidis
Charles H. Swift Distinguished Service Professor
Department of Mathematics
The University of Chicago

The control of the Dyson equation, which is the mean field equation for the eigenvalues of random symmetric matrices, leads to the study of a nonlinear Hamilton-Jacobi-type equation with singularities in the set of probability spaces. In the lecture, I will describe how to establish the well-posedness of the solutions of these equations. Then, I will use these results to obtain large deviations for the eigenvalue of large symmetric matrices and analyze the so-called Iztyson and Zuber formula in mathematical physics. This is part of a joint work with P.-L. Lions and C. Bertucci.