

# Modeling Urban Growth using Fuzzy Cellular Automata

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**Abstract:** Urban modeling is an important tool for efficient policy designing. We herein present a methodological framework for urban modeling which attempts to access the multi-level urban growth dynamics and express them in linguistic terms. The suggested framework incorporates a set of fuzzy systems, each one of which focuses on different aspects of the urban growth dynamics, while the systems' structure and connection provides a work flow closer to the human conceptualization of the phenomenon. Cellular automata techniques are incorporated in the main system's inference engine. The proposed modeling structure has been applied and calibrated in Mesogia – Athens.

**Keywords:** Urban Growth, Rule-based Modeling, Cellular Automata, Fuzzy Logic, Mesogia Athens

## 1 INTRODUCTION

The term urbanization refers to the process of a population changing from rural to urban. Urban growth is a global phenomenon, it is a cause for and an effect as well, of various ecological and socioeconomic processes that take place in both natural and human environments. Urbanization has been occurring in human history under various forms and is an inherent process of human evolution, both physical and cultural. In a social point of view, it could be described as a spatially referenced trade off between different types of human needs and expectations.

For the first time in human history, more than 50% of the global population lives in urban areas while urban population growth rates are increasing. What increases even faster is the urban society's need in space, services, facilities and energy. Considering the above, it is crucial that the urban growth phenomenon is studied and comprehended in order to be directed towards controlled and sustainable patterns and scales of evolution. It is essential to have a clear vision of sustainable urbanization patterns in order to be able to design policies with a significant impact towards sustainability. Such knowledge is necessary for the optimal use of space, services, facilities and energy, which will lead to maximizing the benefits for urban population while minimizing both economical and environmental cost.

A model is a simplified and hence a more comprehensible conceptualization of a real phenomenon or process. To model is to subtract by reality until the final concept is simple but also retains its realism. Nevertheless, simplifying is not simple, especially when it comes to complex phenomena where there lies great inherent spatial, temporal and decision-making heterogeneity (Cheng, 2003). This is a result of socio-economic and ecological heterogeneity itself.

Urban growth is even more difficult to model, since not only it evolves in time but also spreads in space in both continuous and discontinuous ways. That means that apart from the difficulties of studying a spatial phenomenon, when studying urban growth we come across first-seen qualitative interactions, for which we lack the ground to model in a mathematical way. That is because our knowledge, either empirical or data-driven, is merely describing the part of the urban growth dynamics that have already occurred and have been observed rather than the urban growth dynamics in general. What is more, knowledge about the operational scale(s) of urban form and process, and the interaction and parallelism among different scales, is poor (Dietzel, 2005). Urban growth evolves in areas where it has already occurred (vertical expansion) and also extends its borders continuously by sprawling or occurs in segregate areas (horizontal expansion). Moreover, its underlying patterns of evolution are evolving in time as well. This means that even if clear vision of the patterns occurred is acquired it can only be used under certain assumptions of relative stasis.

We should not consider models as calculators; their scope is not only to provide estimations of the future status but also knowledge of the phenomenon. The knowledge derived by a model is as much important as the model's consistency to the real world; knowledge that is meant to be communicated. To resolve these issues we designed a modeling structure that may use spatio-temporal rules, deterministic or stochastic, empirical, data-driven or both. Such a model allows the user to overcome the limitations of the available data by using exogenous knowledge adapted to the model by empirical similarity patterns, providing thus better fit to the real process. The knowledge of this model is expressed in common language describing simple relations with linguistic terms. That provides a conceptualization of the phenomenon and an information flow closer the human conceptualization and makes the model friendly and easy to use, especially to the non-expert users. We propose a structure of fuzzy systems that may be serial, parallel or encapsulated to each other. Apparently the model's structure and the systems' connection are tightly related to the model's efficiency, the work flow and the interpretability of the extracted knowledge. Each system deals with certain aspects of the overall dynamics, while the system that estimates the evolution of the urban growth incorporates cellular automata (CA) techniques. Both CA and fuzzy logic are rather briefly introduced in the next section.

## **2 METHODS INCORPORATED**

In Cellular Automata (CA) the system under study is divided into a set of cells with each cell interacting with all other cells belonging to predefined neighborhoods through a set of simple rules (Krawczyk, 2003). The interactions take place in discrete time steps with each cell's state at any time period estimated by considering the state of the neighboring cells. This approach is repeated continuously in a self-reproductive mechanism with no external interference. Growth is thus simulated through a bottom up approach and this makes CA an appropriate technique for simulating complex phenomena that is difficult to model with other approaches.

In Fuzzy Logic, variables consist of partially overlapping qualitative fuzzy sets. Each fuzzy set is described by a linguistic variable familiar to its quality while quantitative (numerical) information is appointed to the proper fuzzy sets by the correspondent membership function. The knowledge base is represented as linguistic "IF... THEN" rules, connecting hypotheses to conclusions through a certainty factor. The most frequently used inference engines are divided into the stages of aggregation,

implication and accumulation (Kirschfink 1999, Hatzichristos 2001). Aggregation returns the fulfillment of the hypothesis for every rule individually. Implication combines aggregation's result to the rule's certainty factor (CF) resulting to the degree of fulfillment for each rule's conclusion. Finally accumulation corresponds to compromising different individual conclusions into a final result. Fuzzy logic combined to CA provides a proper framework for expressing and mapping the urban growth dynamics.

### 3 ARCHITECTURE OF THE MODEL

The goal is to develop a model transferable to both data rich and data poor applications that simulates in a realistic way the urban growth evolution while it allows the communication of its knowledge. The architecture of the model (*figure 1*) attempts to compromise three contradictious objectives. These are to:

- reproduce efficiently the underlying spatio-temporal patterns of the urban growth dynamics,
- express in comprehensible linguistic terms the knowledge acquired and
- sustain a generic form of the model that is disengaged from data limitations.

Initially, a set of qualitative fuzzy systems processes the input data, both fuzzy and crisp, and estimates certain thematic indexes for the suitability of urbanization in an area. Land-use suitability modeling is an analytical process that determines the fitness of a given unit of land for a specified use (Carr, 2007). The themes that can be used depend on data availability and are such as physical (slope, soil), natural (land use), and accessibility (road networks). These systems conclude over fuzzy variables consisting of three ordered fuzzy symbols (low, average, high) and depict the suitability to urbanization of each area concerning certain points of view – certain thematic suitability maps that are the first intermediate results of the model. The thematic indexes calculated are divided into static (i.e. physical) and variable in time (i.e. accessibility), with variable thematic indexes potentially altered in subsequent stages if certain criteria are met.

These systems use a Mamdani inference engine and further incorporate the use of a new accumulation operator based on the Dempster-Shafer (DS) theory of evidence (Ahmadzadeh, 2001). The basic notion of this operator is that as more rules lead to the same conclusion, the possibility that the conclusion is false decreases. The advantage of the DS operator is that it takes into account not only the strength of each result but also the number of rules that conclude to this specific result. Its disadvantage on the other hand is that it tends to return higher values as the number of rules increases and hence it takes a finer tuning to work efficiently. Moreover this operator should apply to rules with relatively independent hypotheses.

The thematic fuzzy systems run only once and introduce four advantages in the model. First of all they provide useful intermediate results about the urban growth potentials. What is next, they result into decreasing the number of variables that are used in the following systems, making thus easier the further analysis and allowing us to use more simple and hence more comprehensible rules. For modeling, simplification is both necessary and useful (Ness, 2000). Each thematic system uses such aggregation and accumulation operators that focus into the diversifications of the input variables. As a result the outputs – the thematic indexes – tend to be less correlated to each other than the input variables. Finally, it is much more convenient to update the (variable) thematic indexes rather than the initial variables themselves.

Following, the area of study is divided into three fuzzy sets – static non urban, dynamic non urban and urban. Actually the static set is a crisp one, nevertheless is banally considered to be fuzzy. Static areas, as implied by their name, do not alter their status during any stage of the model. If urban density data are available, urban areas are managed by a quantitative fuzzy system (Sugeno inference)

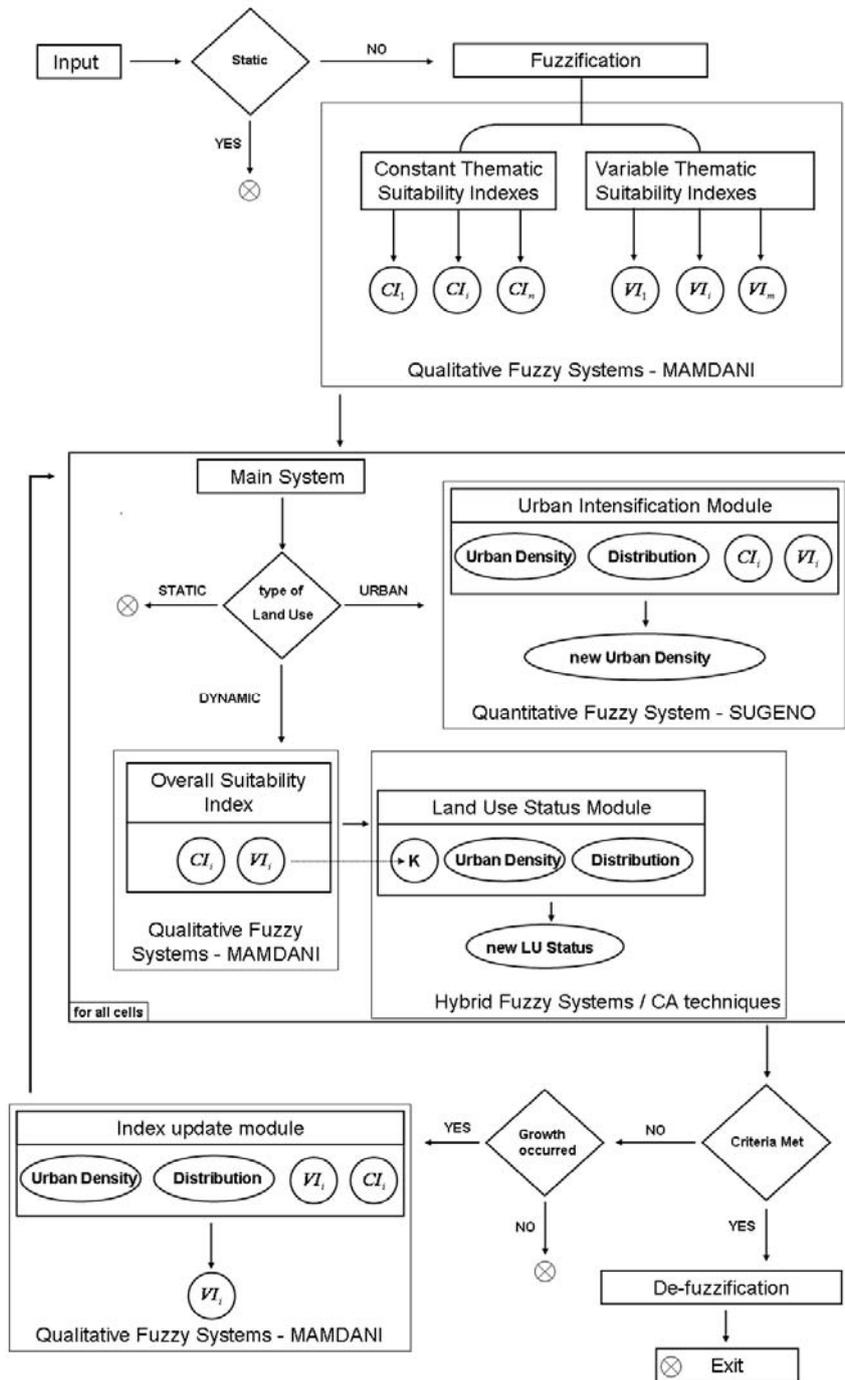


Figure 1: the proposed modeling structure

which simulates the vertical urban growth by estimating the evolution of urban density in urban areas based among others on the distribution of the urban density. When it comes to dynamic areas, a Mamdani system, whose input is the fuzzy thematic suitability indexes, concludes over the overall suitability to urbanization which consists of two fuzzy symbols (low-high). This is the last intermediate result of this model. The suitability systems do not incorporate CA techniques. Once the overall suitability is calculated, a hybrid qualitative fuzzy system that incorporates CA techniques is used to calculate the next status of each area. CA use Moore neighborhood of various radius.

This hybrid system uses a Mamdani inference engine but also incorporates a new non-linear implication operator whose use results to a new form of rules. The linguistic syntax of the rule is the same; the computative difference yet is that this operator raises the current membership value of the conclusion premise in the power of the hypothesis complement. Given the fact that membership values, certainty factors and aggregation results are bounded in  $[0,1]$ , if the hypothesis of the rule is not met at all, such a rule results to no change in the fuzzy set of the conclusion; it returns the initial membership value. On the other hand if the hypothesis is fully met and the rule is deterministic, it results to a certain conclusion – a membership value 1. In any other case it returns a membership value within  $(m,1)$  where  $m$  is the initial membership value. This type of rule is used to increase the certainty of a result for a specific cell as this result is produced again by the same rule that is fired in a subsequent step. Once the whole area has been processed, a last qualitative system is applied in the areas where growth (either expansion or intensification) has occurred, in order to update the variable thematic suitability indexes. This module, as well as the quantitative Sugeno system, has not yet been developed.

This is an iterative procedure that takes place until exit criteria are met. Currently the exit criterion used is the projected urban amount allocated which appears to introduce dependencies between the two types of possible error. These are overestimation error and underestimation error; either a rural cell is considered urban or vice versa. That means that, more or less, for each rural cell that is mistakenly considered urban, there is another urban cell that is mistakenly considered rural. This was partially resolved by introducing a tolerance in the exit criterion; nevertheless, this criterion needs further consideration.

Spatial variability is gained by expressing the rules' certainty factors as functions that take into consideration a spatial 2-D fuzzy variable which expresses the relative location of a cell within the study area. This is applicable to all the systems incorporated, yet it is preferable to introduce spatio-variability in the last possible system of the structure that produces the desirable effect to the results. Temporality may as well be introduced by using a temporal (1-D) variable; at this point though the specific qualifications for doing so, are not clear.

Recently a few fuzzy approaches have been proposed (Dragicevic 2004 and Liu 2001) that introduce single fuzzy systems in CA based urban modeling. They both propose the use of transition functions that are influenced by the various input variables. Dragicevic uses a single variable function for each type of transition (from a certain land use to another) while Liu uses a set of predefined functions, only one of which is applied in each cell for a single step. These approaches resemble to the Takagi-Sugeno approach. The key concept in those approaches is familiar to the model we herein propose but there are many differences and technical advantages. First of all we propose a structured set of fuzzy systems (one of which is hybrid) instead of a single system; that way we may analyze separately the effect of various parameters to the urbanization process. What is more, we emphasize on the specialized use of fuzzy algebra and suggest some new operators so that we can deal with the urbanization process in both a qualitative and a quantitative way. We incorporate simple rules to improve the knowledge interpretability and introduce fuzzy hedges and spatial variability for more efficient simulation and more accurate description of the urban growth patterns. Finally, instead of predefined (Liu) or undefined (Dragicevic) rules we developed a transferable generic rule-extraction module that is used to produce an initial knowledge base.

## 4 KNOWLEDGE EXTRACTION

The knowledge extraction module for the suitability systems incorporates the following approach. First we calculate the percentage of urban cells that belong to each fuzzy set of each input variable. Then we divide the value returned for each set with the percentage of all cells (urban or not) that belong to this fuzzy set. Finally we normalize these values for each single variable separately. This approach is based on the assumption that the more often an attribute (i.e. a certain land use) appears in the study area, the more possible it is to overestimate the actual attraction of urbanization by this attribute. A more sophisticated approach was also considered. This is to use not only single attributes, but combinations of attributes (fuzzy sets intersections) as well (as in Cuesta, 2003). In that case the resolution of the model would be improved, but it would also affect the simplicity and hence the interpretability and the generality of the knowledge base. The benefit for keeping the rules simple is that if another variable is to be added, then all we need to do is add the rules that describe its own effect to the phenomenon, without altering any of the initial rules.

This method produced an initial rule base to which spatio-temporal parameters and fuzzy hedges were further applied through calibration process. A very important attribute of the extracted knowledge is that it can be easily changed according to empirical similarity patterns, common logic or simple observations in order to include information that is missed by the available data.

In the hybrid fuzzy system we use simple empirical or common sense rules that are also subjected to a calibration process. Due to the (in general) irreversible nature of CA, data for two time-points may depict the result of the CA process but not the dynamic or the specific form of its transition engine; in such a system, simulation is the only way to predict outcomes (Clarke, 1997). At this stage, having already reduced the number of variables that are used, it is relatively easy - and even kind of fun in a more literal sense of Urban Gaming Simulations (Cecchini, 2001) - to experiment with each rule's syntax and its parameters fine tuning.

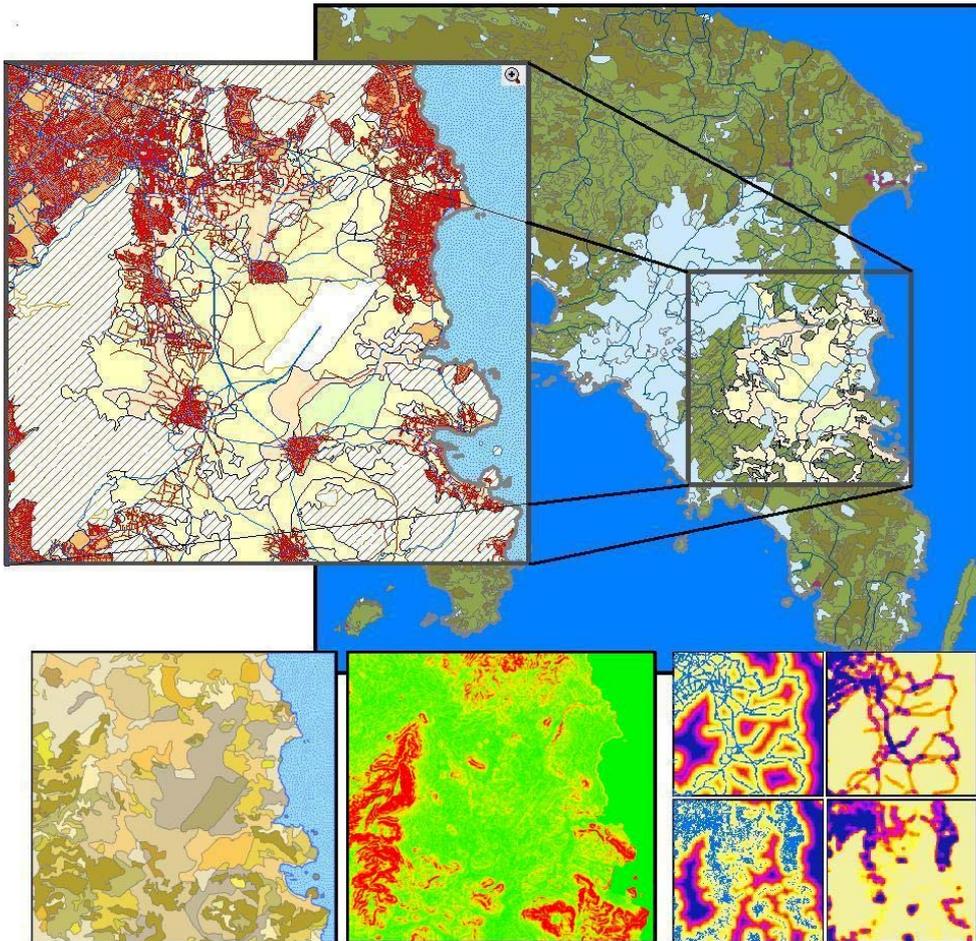
## 5 CASE STUDY

Our case study took place in Mesogia, in the east area of Attica. This is a mainly rural area that has developed rapidly during the last 15 years as a result of the new airport and the construction of a new highway. The available data set includes land use (Corine database) for 1994, 2001 and 2004, the road network and a DEM of the area. Road network data were separated in primary and secondary and layers of distance and density were derived. The period 1994-2001 was used in order to extract a knowledge base which was applied in the 2001-2004 period and checked for its fitting to the real world. The location of the area and the available crisp data are shown in *figure 2*.

For a fuzzy knowledge base to be both efficient and comprehensible fuzzy sets should be carefully defined while its number should be the least that adequately expresses qualitative diversifications of the objects. In our case the fuzzy variables that are linked to numerical crisp variables use three fuzzy sets. The thematic suitability variables consist of tree symbols as well, while the overall suitability and urban status variables consist of two symbols. Finally the land use data are represented as singletons.

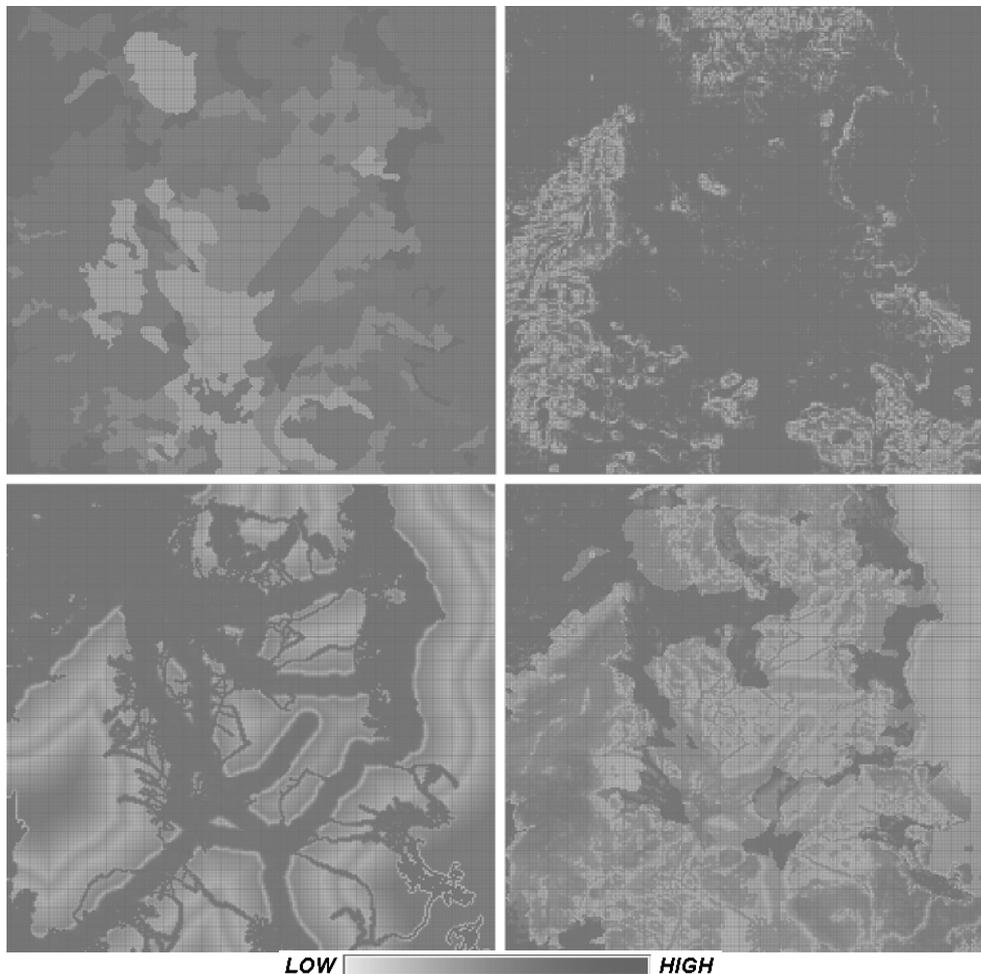
The first results indicated a spatial heterogeneity in the error. This was caused by the spatially heterogeneous effect of the land use suitability to the overall suitability which was further passed to the final error through the effect of the overall suitability to the urbanization process. This problem was resolved by introducing the spatial fuzzy variable in the land use suitability fuzzy system. Spatial variability was also introduced in the knowledge base of the hybrid system, in order to use

neighborhoods of various radiuses. The reason to do so lies in the CA technical characteristics. In CA, the ‘Speed of Light’ (Itami, 1994), which is the maximum speed of information propagation, is limited by the radius of the neighborhood used. That means that in areas where urbanization spreads faster we need to use wider neighborhoods; otherwise significant underestimation error is occurred. An alternative approach would be to search for qualitative measures that distinguish those specific areas from the others, but this was not possible at this level of analysis on this rather poor data set.



*Figure 2:* location of the study area (top) and available crisp data (bottom from left to right) – Corine Land Cover, DEM, distance from and density of primary and secondary road network

Following, after the application of fuzzy hedges, the final knowledge base was formed and approved to describe the urban growth dynamics that took place in the study area during the period 1994-2001. Once the analysis was completed, the knowledge base derived by analyzing the 1994-2001 growth patterns was used to estimate the 2004 urban status. The intermediate suitability maps are shown in *figure 3*.



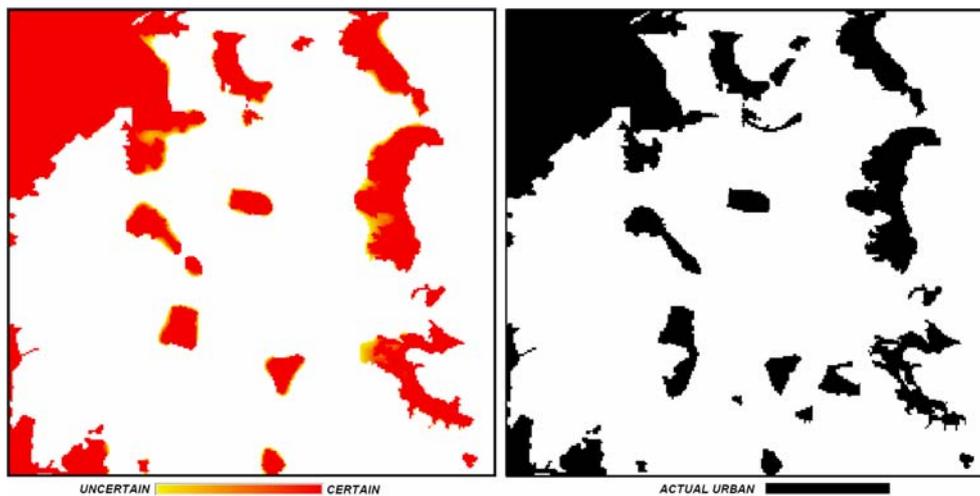
*Figure 3:* mappings of the intermediate suitability results, from left to right, top – Land Use and DEM, bottom - accessibility and overall suitability

In order to validate the model's behavior and assess the fitting of the results to real data, three indicators were defined. The indicators may refer to the whole study area (map error) or the area where growth occurs, either in the real world or in the model (model error). Urban modeling literature suggests a variety of sophisticated error indicators. At this time though, we use very simple and easy to calculate and understand indicators that are defined as follows:

- U(nderestimation) : the percentage of cell's that were mistakenly considered to become urban,
- O(verestimation) : the percentage of cell's that were mistakenly considered to remain rural,
- F(ull) : the percentage of cell's whose status was mistakenly considered (either urban or rural).

These indicators were calculated both for the period of analysis (1994-2001) and the test-application period (2001-2004). These two periods are not directly comparable to each other, since they are of different length but also different levels of urban growth occurred during them – the new airport and the new highway were completed in 2001. For further comparisons, the same indicators

were also calculated for the full period 94-04. It must be stated that although the module for updating suitability indexes is not yet developed and that the exit criterion introduces error dependencies – issues to be dealt in the future - the error indexes are reduced, at least in comparison to our previous work (Mantelas et al, 2007). Another important improvement is that the error propagation through iterative steps has been diminished which suggests that this model can provide better long term estimations than the previous. Apart from the numerical values of the error indexes, this model fits better to the pattern of urban cover in the area. *Figure 4* visualizes the estimation of urban cover for the year 2004 based on 2000 and the actual 2004 urban cover. The values of the indicators for each period studied are shown in *figure 6*, while *figure 5* depicts their evolution through the iterative process for the estimation of the 1994-2001 period.



*Figure 4:* estimation of the 2004 urban cover based on data for the year 2001 (left) and error mapping (right)

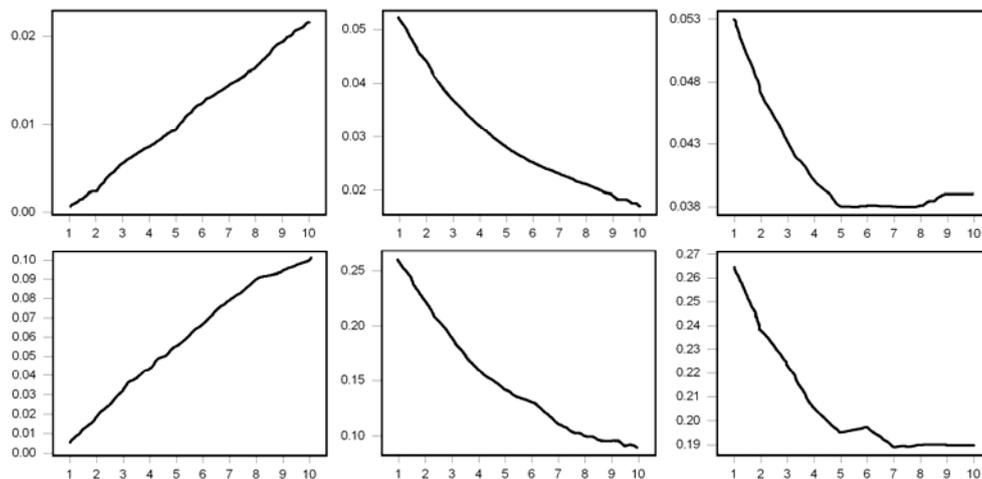


Figure 5: the evolution of the map error (top) and the model error (bottom) over the iterative steps of the model for the period 94-01 - overestimation (left), underestimation (middle) and full error (right).

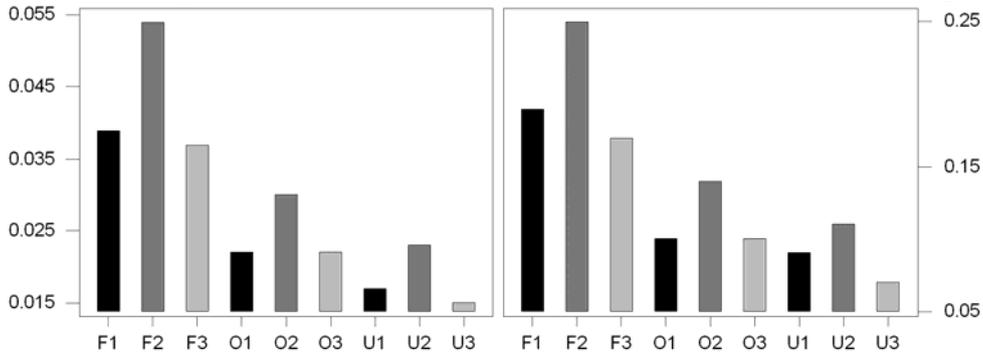


Figure 6: values of the map error indicators on the left and the model error indicators on the right – Full, Over and Under - 1 corresponds to 94-01, 2 to 94-04 and 3 to 01-04 period

## 6 CONCLUSIONS AND FUTURE WORK

The herein proposed modeling structure simulates efficiently the urban growth in the Mesogia for the periods studied. According to the fitting indicators used and under the assumption that urban growth dynamics will not alter significantly this model is capable of providing a vision to the future evolution of the urban growth phenomenon in the area and hence contribute to a sound urban plan.

CA and fuzzy systems intergrade successfully and in some cases naturally. The rule-based CA transition engine can be substituted by a fuzzy inference engine with no particular modifications. What is more, unlike binary CA or binary rule based systems in general, fuzzy systems' theory does not need a separate set of collision rules.

This model inherits the advantages and disadvantages of the CA techniques incorporated. That means that it is very efficient in simulating the spread of existing urban cover but fails to capture the urbanization of detached areas. For that reason, the use of agent-like rules (as in Torren et al, 2005) is considered. The heart of the model, the fuzzy inference engines that are used, provides the proper information management framework and further allows the knowledge expression in common linguistic terms. The herein presented non-linear operators improve the models behavior and will be further developed; both the Dempster-Shafer accumulation operator and the polynomial implication operator introduced in this work are improving the models behavior and despite their need for fine tuning they will remain active in the model. Finally, the fuzzy hedges and the spatial fuzzy variable applied appear to be most effective.

There are several directions to which we need to extend or reconsider our model. Temporal rules are supported by this model but the available data set is not rich enough to allow the extraction of a temporal knowledge base. What is next, the Sugeno and the suitability update modules need to become functional and multi-agent rules to be introduced. Following there are the exit criteria and the error indicators issues that need to be reconsidered. Finally, we need to find more and better data sets to experiment on and fully explore the capabilities of this model.

## BIBLIOGRAPHY

- Ahmadzadeh M., Petrou M., 2001, An Expert System With Uncertain Rules Based on Dempster-Shafer Theory, IEEE International Geoscience and Remote Sensing Symposium, IGARSS, Sydney, Australia.
- Benenson I., Kharbush V., 2006, Geographic Automata Systems and the OBEUS Software for their Implementation, Complex Artificial Environments, Springer Berlin Heidelberg, p.137-153,
- Blecic I., Cecchini A., Prastacos P., Trunfio G.A., Verigos E., 2004, Modelling Urban Dynamics with Cellular Automata: A Model of the City of Heraclion. 7th AGILE Conference on Geographic Information Science, University of Crete Press, Heraklion, Greece,
- Carr H. M., Zwick P. D., 2007, Smart Land-Use Analysis - The LUCIS Model, ESRI Press,
- Cecchini A., Rizzi P., 2001, Is Urban Gaming Simulation Useful?, Simulation Gaming, vol.32, no.4, p.507,
- Cheng J., Masser, I., 2003, Understanding Urban Growth System: Theories and Methods. 8th International Conference on Computers in Urban Planning and Urban Management, Sendai City, Japan.
- Clarke K. C., Hoppen S., Gaydos L., 1997 ,A Self-Modifying Cellular Automaton Model of Historical Urbanization in the San Francisco Bay Area. Environment and Planning B, vol. 24, Planning and Design, p.247-261,
- Cuesta R.C.; Diaz I.; Cuadrado A.A.; Diez A.B., 2003, A Visual Approach for Fuzzy Rule Induction, Proceeding Emerging Technologies and Factory Automation conference, Lisbon, Portugal
- Dietzel Ch., Clarke K. C., 2004, Replication of Spatio-Temporal Land Use Patterns at three Levels of Aggregation by an Urban Cellular Automata. Lecture Notes in Computer Science, vol. 3305, p. 523-532,
- Dietzel Ch., Oguz H., Hemphill J. J., Clarke K. C., Gazulis N., 2005, Diffusion and Coalescence of the Houston Metropolitan Area: Evidence Supporting a New Urban Theory, Environment and Planning B: Planning and Design, vol.32, p.231-246,
- Dragicevic S., 2004, Coupling Fuzzy Sets Theory and GIS-based Cellular Automata for Land-Use Change Modeling, In Fuzzy Information, IEEE Annual Meeting of the Processing NAFIPS'04, Banff, Canada, vol.1, P.203-207
- Hatzichristos Th., 2001, GIS and Fuzzy Logic in Spatial Analysis. Educational notes, NTUA,
- Itami M. R., 1994, Simulating spatial dynamics: cellular automata theory, Landscape and Urban Planning, vol.30, p.27-47,
- Kirschfink H., Lieven K., 1999, Basic Tools for Fuzzy Modeling. Tutorial on Intelligent Traffic Management Models in Helsinki,
- Koutsopoulos K., 2002, Geographic Information Systems and Spatial Analysis. Papatotiriou,
- Krawczyk R.J., 2003, Architectural Interpretation of Cellular Automata, Poster presented at NKS 2003, Boston,
- Liu Y., Phinn S.R., 2001, Developing a Cellular Automaton Model of Urban Growth Incorporating Fuzzy Set Approaches. Proceedings of the 6th International Conference on GeoComputation, University of Queensland, Brisbane, Australia
- Mantelas L., Hatzichristos Th., Prastacos P., 2007, A Fuzzy Cellular Automata Based Shell for Modeling Urban Growth – A Pilot Application in Mesogia Area. 10th AGILE International Conference on Geographic Information Science, Aalborg University, Denmark
- Morshed A., 2002, Land Use Change Dynamics: a Dynamic Spatial Simulation. PhD Thesis,
- Mulianat I., Hariadi Y., 2004, Urban Area Development in Stochastic Cellular Automata. Urban/Regional, EconWPA,
- Ness G. D., Low M. M., 2000, Five Cities : Modelling Asian Urban Population-Environment Dynamics, Oxford University Press, p.43-67,
- Torren P. M., Benenson I., 2005, Geographic Automata Systems. Geographical Information Science, vol. 19, no.4, p.385–412,

Vancheri A., Giordano P., Andrey D., Albeverio S., 2004, A Model for Urban Growth Processes with Continuum State Cellular Automata and related Differential Equations,  
Zadeh L. A., 1965, Fuzzy Sets, Information and Control, no.8, p.338-353.