UNDERWATER SOURCE DETECTION AND LOCALIZATION WITH INCOHERENT DATA

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Abstract: *We consider the problem of imaging the location of one or more sources in a waveguide with random sound speed fluctuations. Our data is the acoustic pressure field measured at an array of hydrophones. Randomness is used to model the effect of internal waves on the sound channel and although the strength of the fluctuations is small, the transmitted signal is significantly affected from the multiple scattering of the waves with the random inhomogeneities, especially since we consider large propagation distances between the source and the receiver array. In such regimes, traditional coherent imaging fails. In an ideal acoustic waveguide sound propagates through guided modes that do not interact with each other. This is no longer true, however, in a random acoustic waveguide for which, under the assumptions of the diffusion and the forward scattering approximations, a stochastic system of differential equations for the mode amplitudes can be derived. This system describes how mode coupling and transfer of energy between modes occurs in a random waveguide. Taking into account this random model, an incoherent method of inversion based on a system of transport equations has been derive in Borcea et al., 2010. In this work we apply such an incoherent method for determining the location of multiple sources. We use a numerical model for a waveguide with a rigid bottom and a mean sound speed profile coming from the YELLOW SHARK '94 experiment to which we add depth and range dependent random fluctuations. We show with simulated data that the inversion method is very accurate, and stable with respect to the realization of the random medium, for estimating the distance between the receiver array and the detected source, even in the case of only two receivers.*

Keywords: *source imaging, random waveguides, incoherent imaging*

1. FORMULATION OF THE PROBLEM

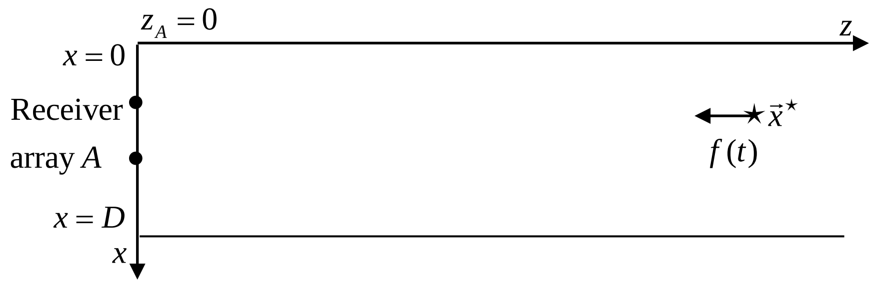

Fig.1: Schematic of the problem setup.

We consider acoustic wave propagation in a two dimensional inhomogeneous waveguide with planar horizontal boundaries (see Fig.1). The acoustic pressure field $p(t,x,z)$ is governed by the scalar wave equation

$$\Delta p(t,x,z) - \frac{1}{c^2(x,z)} \frac{\partial^2 p(t,x,z)}{\partial t^2} = \nabla \cdot \vec{F}(t,x,z).$$

The boundary conditions are those of a pressure release surface $p(t,x,z) = 0$ at the top, i.e., $x = 0$ and of a rigid bottom $\partial p(t,x,z)/\partial x = 0$ at $x = D$. The source term is $\vec{F}(t,x,z) = -f(t)\delta(x - x^\star)\delta(z - z^\star)\vec{e}_z$ and models a point like source at $\vec{x}^\star = (x^\star, z^\star)$, emitting a pulse $f(t)$ towards the array which is located at z_A. Here $\vec{e}_z$ is the unit vector pointing in the $z-$direction. Note that we use a coordinate system with range origin at the array and assume that propagation is from right to left.

We want to model sea water with internal waves caused by changes in temperature and salinity. In such environments the sound speed $c(x,z)$ has an (x,z) dependent fluctuating part (see [1]) that can be modeled by

$$c_0^2(x)/c^2(x,z) = 1 + \varepsilon \nu(x,z). \tag{1}$$

Here $\nu(x,z)$ is an isotropic, statistically homogeneous random process with mean zero and a smooth correlation function. The perturbation parameter ε is small and ranges between $1\% - 3\%$. The effect of the random inhomogeneities on the acoustic pressure field becomes important for waves that travel over long distances in the waveguide.

The inverse problem that we want to solve is to estimate the location $\vec{x}^\star$ of the source, given the observed acoustic pressure field $p^{obs}(t, x_r, z_A)$ at a vertical receiver array A. The observed pressure field could be obtained either by experimental measurements or by numerical simulation, as is the case in this paper. The water column can in principle contain an unknown number of sources, so the first step is to determine their number.

2. THE PRESSURE FIELD IN RANDOM WAVEGUIDES

In our model, Eq.(1), the sound speed in the unperturbed waveguide, $c_0(x)$, depends only on the transverse direction, x. We know that for such velocity profiles energy is transmitted by independent guided modes, the orthogonal eigenfunctions of the symmetric differential operator $\partial_x^2 + \omega^2 / c_o^2(x)$. Let us denote by $\phi_j(\omega, x)$ the eigenfunctions of this operator, by $N(\omega)$ the number of propagating modes and by $\beta_j(\omega)$ the wavenumbers.

According to [2], we can write the Fourier coefficients of the random pressure field recorded at receivers located at (x_r, z_A) in the following form

$$p^\varepsilon(\omega, x_r, Z) \approx \frac{\hat{f}^\varepsilon(\omega)}{2} \sum_{j,l=1}^{N(\omega)} \sqrt{\frac{\beta_l(\omega)}{\beta_j(\omega)}} T_{jl}^\varepsilon(\omega, Z) \phi_j(\omega, x_r) \phi_l(\omega, x^\star) e^{i\beta_j(\omega) Z/\varepsilon^2},$$

where we introduced the scaled range $Z = \varepsilon^2(z^\star - z_A)$. Here $T^\varepsilon(\omega, Z)$ is the transfer or propagator matrix (i.e., the fundamental solution) of the system of linear stochastic differential equations with unknowns the random amplitudes of the modes. This system describes the coupling of the modes by cumulative scattering in the random medium over very long ranges (in the diffusion approximation). To obtain this expression, in [2], it is also assumed that the coupling between the forward and backward propagating modes becomes negligible as $\varepsilon \to 0$ (forward scattering approximation). In the asymptotic limit $\varepsilon \to 0$, the expectation of the transfer matrix is given by

$$\lim_{\varepsilon \to 0} E[T_{jl}^\varepsilon(\omega, Z)] = \delta_{jl} e^{-\frac{1}{\mathsf{S}_j(\omega)} Z + i \frac{1}{\mathsf{L}_j(\omega)} Z},$$

where δ_{jl} is the Kronecker delta symbol. The scales $\mathsf{S}_j(\omega)$ and $\mathsf{L}_j(\omega)$ are defined in [3, Eqs. (3.28) and (3.31)] and depend on the frequency and the correlation function of the random fluctuations. The scattering mean free path $\mathsf{S}_j(\omega)$ indicates the range scale over which the field becomes incoherent. Let us denote by S_1 the scattering mean free path for the first mode. For arrays at distances $L \geq \mathsf{S}_1$ from the source, the coherent part of the mode amplitudes decay exponentially in range and the random fluctuations become dominant for all modes.

3. THE COST FUNCTION FOR THE RANGE ESTIMATION

For imaging we first compute the projection of the Fourier coefficients of the acoustic pressure field onto the waveguide modes $\hat{P}_j(\omega, z_A) = \int dx_r \, \hat{p}(\omega, x_r, z_A) \phi_j(\omega, x_r)$. We then obtain the ``dispersion function''

$$R(\zeta, j) = \int_{|\omega - \omega_o| \leq 2\pi B} \frac{d\omega}{2\pi} \int_{|\omega - \omega'| \leq 2\pi \Omega_d} \frac{d\omega'}{2\pi} \hat{P}_j(\omega, z_A) \overline{\hat{P}_j(\omega', z_A)} e^{-i(\beta_j(\omega) - \beta_j(\omega'))\zeta},$$

by cross-correlating $\hat{P}_j(\omega, z_A)$ with $\hat{P}_j(\omega', z_A)$ over frequency windows of support Ω_d. We denote by R^{obs} the dispersion function for the data and by R^M the theoretical expression that we obtain by using our asymptotic model.

We compute R^M (see [1, Eq. 6.13] and [2, proposition 20.7]) via the Wigner distribution $\left\{ W_j^{(l)}(\omega, \tau, z) \right\}_{j=1,\dots N(\omega)}$. For $z > 0$, it satisfies the following system of transport equations

$$\left[\frac{\partial}{\partial z} + \beta'_j(\omega) \frac{\partial}{\partial \tau} \right] W_j^{(l)}(\omega, \tau, z) = \sum_{n \neq j} \Gamma_{jn}^c(\omega) \left[W_n^{(l)}(\omega, \tau, z) - W_j^{(l)}(\omega, \tau, z) \right],$$

with initial condition $W_j^{(l)}(\omega,\tau,0)=\delta(\tau)\delta_{jl}$ and $\delta(\tau)$ being a Dirac delta distribution. This system describes the transfer of energy between the modes.

The solution of this system can be computed in the Fourier domain and is given by $\hat{W}_j^{(l)}(\omega,h,Z)=[e^{(ih\mathrm{B}'(\omega)+\Gamma^c(\omega))Z}]_{jl}$, with $\mathrm{B}'(\omega)=diag(\beta'_1(\omega),...,\beta'_N(\omega))$, a diagonal matrix and $\beta'_j(\omega)$ the derivative of $\beta_j(\omega)$ with respect to ω. The matrix $\Gamma^c(\omega)$ is defined in [2, section 20.3.1, Eqs. 20.49-20.52], depends on the frequency and the correlation function of the fluctuations and models the mode coupling. Let us denote by $\Lambda_2(\omega)$ the second eigenvalue of $\Gamma^c(\omega)$. We can introduce now the equipartition distance $\mathsf{L}_{equip}(\omega)=-1/\Lambda_2(\omega)$, the range scale over which the energy of the wave becomes distributed uniformly over the modes (see [2, section 20.6.2]).

The unknown parameters (Z,α,ℓ) can be estimated by minimizing the following cost function over (Z^s,α^s,ℓ^s)

$$O\left(Z^S,a^S,\ell^S\right)=\sum_{j\in S}\int d\zeta\left|\frac{R^{obs}\left(\zeta,j\right)}{R^{obs}\left(\zeta_j,j\right)}-\frac{R^M\left(\zeta,j;Z^S,a^S,\ell^S\right)}{R^M\left(\zeta_j^M,j;Z^S,a^S,\ell^S\right)}\right|,$$

where $R^{obs}(\zeta_j,j)=\max_\zeta\left|R^{obs}(\zeta,j)\right|$ and $R^M(\zeta_j^M,j;Z^s,\alpha^s,\ell^s)=\max_\zeta\left|R^M(\zeta,j;Z^s,\alpha^s,\ell^s)\right|$. The set S of the modes is determined by $R^{obs}(\zeta_j,j)>\delta$, with δ a user defined tolerance.

4. SIMULATION OF THE EXPERIMENT

We obtain the observed field $p^{obs}(t,x_r,z_A)$ (data) by computing numerically the solution of the time dependent wave equation using the finite element method described in [4] coupled with PMLs on the vertical boundaries of the waveguide. Let us denote by $\hat{p}^{obs}(\omega,x_r,z_A)$ the Fourier transform (in time) of the data. The source excitation used is a Gaussian pulse with bandwidth $50-950\mathrm{Hz}$ and central frequency $f_c=500\mathrm{Hz}$. For the inverse problem we will consider data in sub-bands $[f_o-B,f_o+B]$ for $B=62.5\mathrm{Hz}$, $\Omega_d=14.8\mathrm{Hz}$ and different values of f_o.

We use a depth dependent sound speed profile coming from the YELLOW SHARK '94 experiment in the South Elba, in Italy, cf. [5]. We take fluctuations of the sound speed as in Eq.(1) with $\varepsilon=3\%$, using a Gaussian correlation function $E[\nu(x,z)\nu(x',z')]=\alpha e^{-(|x-x'|^2+|z-z'|^2)/2\ell^2}$. The correlation length is $\ell=0.5\lambda_c$ and the amplitude is $\alpha=1\mathrm{m}^{-2}$ ($\lambda_c=c_o/f_c$ and $c_o=1500\,\mathrm{m}/s$). In the sequel, we invert also for ℓ and α. The depth of the waveguide is $D=20\lambda_c$ and our data are computed on an array with 201 equidistant receivers, (full aperture case). We have two unknown sources, one is at $\vec{x}_1^\star=(15\lambda_c,393\lambda_c)$ and the other is at $\vec{x}_2^\star=(5\lambda_c,493\lambda_c)$. The direct arrival from the source at $\vec{x}_2^\star$ to the array is at time $z_2^\star/c_o=0.986\mathrm{s}$ and the data are computed in the time window $t\in(0,6.3306)\mathrm{s}$.

Note that for $f_o=350\mathrm{Hz}$ or $400\mathrm{Hz}$, S_1 : L_{equip} : $300\lambda_c$ and for $f_o=887\mathrm{Hz}$, S_1 : $50\lambda_c$ and L_{equip} : $200\lambda_c$. The distance between the array and the source which is closer to it is

$393\lambda_c$. Therefore, for $f_o \geq 350\text{Hz}$ our data are incoherent and coherent methods do not work.

5. RESULTS

We present in this section range estimation results, for the test case described in section 4. In all our plots the correct values of the parameters are indicated with a circle.

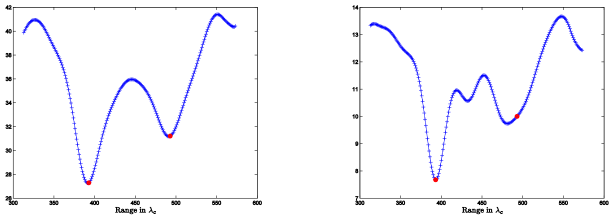

Fig.3: First pass of range estimation for full array aperture on the left and using 2 sparse receivers on the right, $f_o = 887$ Hz.

In Fig.3 we plot $O(Z^s, \alpha^s, \ell^s)$ for the optimal values $\alpha^s = \alpha^{opt}$ and $\ell^s = \ell^{opt}$ at $f_o = 887\text{Hz}$ over a range interval that contains both sources. We show results for one realization of the random medium. However, similar results have been obtained for other realizations. Full array aperture is used for the result on the left and only 2 sparse receivers located in the middle of the waveguide, at $9.8050\lambda_c$ and $10.1950\lambda_c$ are used for the right plot. The two minima we observe, indicate that we have two sources. Therefore, we can locally compute $O(Z^s, \alpha^s, \ell^s)$ around these two peaks to determine the location of each source more accurately.

In Fig.4 we show cross-sections of $O(Z^s, \alpha^s, \ell^s)$ over shorter range intervals that contain each source, separately. In each plot we fix two parameters at the optimal values and display the variation in the third parameter. This incoherent range estimation approach is very robust and gives very good results even when we significantly decrease the array aperture.

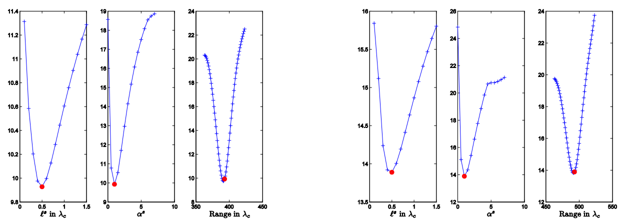

Fig.4: Full array aperture results, for $\vec{x}_1^\star$ (left) and $\vec{x}_2^\star$ (right), with $f_o = 887$ Hz.

The performance of our approach when we use only two receivers is illustrated in Fig.5. The results concerning $\vec{x}_1^\star$ with $f_o = 887\text{Hz}$ (the highest available frequency in our data) are shown on the left plot of Fig.5 and are very good. For the source that is located further away from the array ($\vec{x}_2^\star$) we do not obtain a very precise estimate at $f_o = 887\text{Hz}$ but our estimate is accurate, if we lower the frequency to $f_o = 400\text{Hz}$ (see Fig.5-right).

Our results illustrate that we have a convex functional and therefore the minimization process is easy. In general, our cost function has a clear minimum close to the correct values of the unknown parameters. The values of α and ℓ can be slightly off but this does not significantly affect the range estimation. Here we use a Gaussian model for the correlation function of the medium and seek the parameters α and ℓ. Similar results have been obtained when we replace the Gaussian by other correlation functions, such as an exponential, see [1].

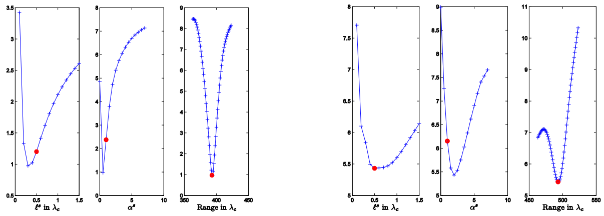

Fig.5: Range estimation results using only two receivers. For $\vec{x}_1^\star$ *with* $f_o = 887\,Hz$ *(left). For* $\vec{x}_2^\star$ *with* $f_o = 400\,Hz$ *(right).*

6. CONCLUSIONS

In this work we exhibited the performance of the incoherent approach proposed in [1], for multiple source localization in a realistic sea acoustic waveguide with random inhomogeneities in the bulk medium. We have seen that the incoherent approach for the range estimation of the source is very robust and it gives accurate and reliable results using only two receivers, even in the case where the distance between the source and the array is beyond the equipartition distance. In this paper, we considered only range estimation. As shown in [1], the estimation of the source depth is more delicate (work in progress).

7. ACKNOWLEDGEMENTS

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REFERENCES

[1] **L. Borcea, L. Issa, and C. Tsogka**, Source localization in random acoustic waveguides, *SIAM Multiscale Modeling Simulations*, 8(5), pp. 1981-2022, 2010.

[2] **J. P. Fouque, J. Garnier, G. Papanicolaou, and K. Sølna**, *Wave propagation and time reversal in randomly layered media*, Springer, New York, pp. 545-597, 2007.

[3] **L. Borcea, J. Garnier, and C. Tsogka**, A quantitative study of source imaging in random waveguides, *Commun. Math. Sci.*, 13(3), pp. 749-776, 2015.

[4] **E. Bécache, P. Joly, and C. Tsogka**, An analysis of new mixed finite elements for the approximation of wave propagation problems, *SIAM J. Numer. Anal.*, 37(4), pp. 1053-1084, 2000.

[5] **J. P. Hermand and P. Gerstoft**, Inversion of broad-band multitone acoustic data from the YELLOW SHARK summer experiments, *IEEE J. Ocean. Eng.*, 21(4), pp. 324-346, 1996.