A hybrid approach for ocean acoustic tomography in range dependent environments based on statistical characterization of the acoustic signal and the identification of modal arrivals

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Summary
A hybrid approach for problems of ocean acoustic tomography is presented, based on the statistical characterization (SC) of the acoustic signal followed by a mode identification and linear inversion. The statistical characterization is used for the estimation of a reference solution to the inverse problem of estimating the sound speed profile in the water column. This non-linear inversion problem is solved using a Genetic Algorithm. By applying first order perturbation approach, variations of the sound speed profile are associated with modal travel time variations. This relationship provides the framework for the development of an iterative linear inversion scheme which converges when the reference environment is close to the actual one and provides a fine tuning of the results obtained by the non-linear inversions. The performance of the method is demonstrated by means of a simulated experiment in range-dependent environment representing a cold eddy.

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1. Introduction

This paper deals with a hybrid inversion scheme for the estimation of the sound speed profile in shallow water environment, based on the exploitation of an acoustic signal received at a single hydrophone. The signal is initially characterized using a statistical approach which, with the help of a Genetic Algorithm (GA) leads to an initial estimation of the environmental parameters under consideration [1]. The inversion results are further improved by means of a linear inversion scheme, based on mode identification and the use of a linear sensitivity kernel introduced by Rajan et al [2] and thereafter applied for tomographic inversions either as a stand alone inversion [3], [4] or in a second phase of a hybrid inversion scheme [5]. Actually the scheme suggested here is similar to the one presented in [5] , with the only difference being the first phase, which, instead of applying a matched modal arrivals scheme, a statistical characterization scheme associated with the GA is applied.

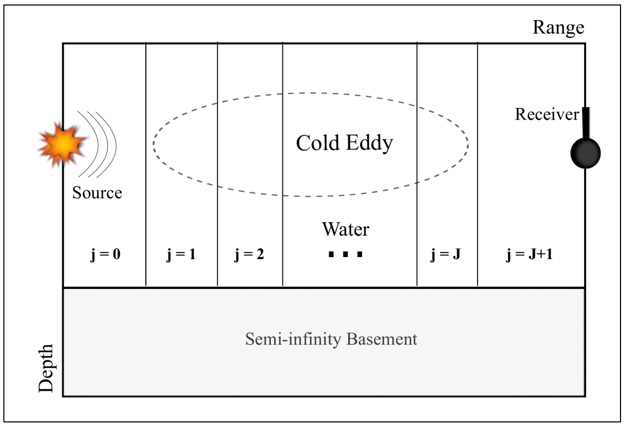

Figure 1. The shallow water environment with the cold eddy.

The scheme is tested here for a shallow water environment of axial symmetry with flat bottom including a cold eddy in the water column as illustrated in Figure 1. The local variation of the temperature of the water at some region of the environment is described by a variation of the corresponding sound speed profile which is of range-dependent character but of compact support. This case is very interesting for typical applications of acoustical oceanography, when oceanographic structures of 3-D character must be estimated. We will further assume that the sound speed structure of the cold eddy at a vertical slice is represented by means of M orders of Empirical Orthogonal Functions (EOFs) $f_m(z)$, which are the basis functions upon which the deviation from the reference sound speed profile $c_0(z)$ is built, according to formula (1)

$$c(z,r) = c_0(z) + \sum_{m=1}^{M} a_k(r) f_m(z), \tag{1}$$

where $R_1 < r < R_2$, and R_1 , R_2 denote the estimated range of beginning and end of the eddy.

In the real world, the EOFs are obtained by analyzing historical data representing typical anomalies in the water column. In order to assess the range-dependent structure of the eddy an appropriate discretization in range should be applied so that the inverse problem is expressed as a discrete one.

Following exactly the same procedure as in [5], J segments of equal width are considered to represent the eddy. In each of these segments the sound speed profile is given by the following formula :

$$c_j(z) = c_0(z) + \sum_{m=1}^{M} a_{j,n} f_m(z), \quad j = 1, \times, J. \tag{2}$$

The coefficients $a_{j,n}, j = 1, \ldots, J, m = 1, \ldots, M$, are the unknown parameters to be recovered and define the vector $\mathbf{m}$. Thus, the total unknowns of the inversion scheme are $M \cdot J$.

The bottom structure, which extends to infinity is considered known.

2. The hybrid scheme

2.1. First part : Statistical characterization and non-linear inversion scheme (SCS)

Figure 2 presents a typical reception of a tomographic signal. In fact it corresponds to the simulated signal to be considered as the actual signal in the test case to be presented later. It is obtained by using the Coupled Normal Mode program MODE4 to calculate the system transfer function $H(\mathbf{x_r}, \mathbf{x_s}; \omega)$ where $\mathbf{x_r}$ and $\mathbf{x_s}$ are the source and receiver positions vectors respectively. The system transfer function is thereafter multiplied by the source excitation function $S(\omega)$, and by means of an Inverse Fourier transform we get the signal in the time domain.

$$p(\mathbf{x_r}, \mathbf{x_s}; \omega) = H(\mathbf{x_r}, \mathbf{x_s}; \omega) S(\omega) \tag{3}$$

$$S(\mathbf{x_r}, \mathbf{x_s}; t) = \mathcal{F}^{-1}\{p(\mathbf{x_r}, \mathbf{x_s}; \omega)\} \tag{4}$$

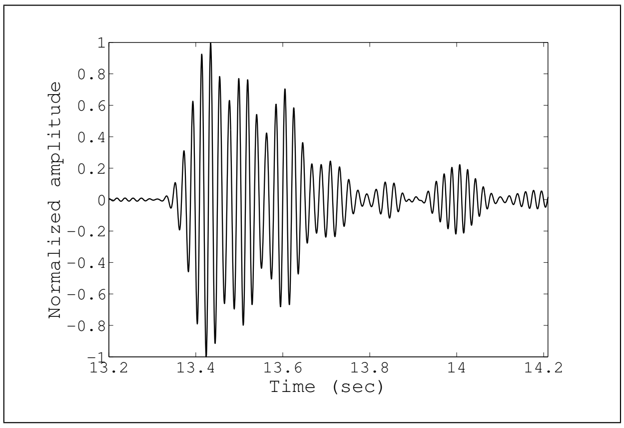

Figure 2. The simulated signal

The source excitation functions is taken to be of Gaussian type while source range is taken to be 0 ($x_r = 0$).

The signal is characterized using the statistics of the wavelet sub-band coefficients as suggested in [1]. In previous works it has been shown that the wavelet coefficients of a typical tomographic signal, here simulated by means of a Gaussian excitation function, obey a Symmetric Alpha Stable distribution (SaS) defined by a pair of the parameters α, γ.

Thus, for an L-level wavelet analysis, the signal can be characterized by L detailed and 1 approximation coefficient vectors, each one of which consisting of only two elements. Hence the signal feature is represented by a vector $\mathbf{d}$ as following:

$$S \leftrightarrow \mathbf{d} = (\alpha^0, \gamma^0, \alpha^1, \gamma^1, \ldots, \alpha^L, \gamma^L), \tag{5}$$

It was also shown that $L = 3$ is an adequate limit of the multilevel analysis of typical underwater tomographic acoustic signals. Thus, the observables of a typical reception of a tomographic signal consists of a vector of 8 elements.

Given the underlying acoustic propagation problem defined through the functional T, the inverse problem is expressed as following :

$$T(\mathbf{d}, \mathbf{m}) = 0 \tag{6}$$

The inverse problem thus defined, is non-linear and is amenable to optimization inversion schemes, based on repeated simulations of the received signal for a class of candidate environments and the use of a cost-function which is appropriate for the nature of the observables. Here, the fact that the actual observables are statistical distributions, an efficient cost-function is the *Kullback-Leibler Divergence* (**KLD**) [6] which it has been used in previous works [1].

2.2. Second part : Local search scheme (LS)

The scheme described above is not based on any specific physical observable as it is the case of traditional inversion schemes. This is actually its basic advantage. However, when physical observables can be defined in the recorded signal, alternative inversion schemes can be applied. The idea of a hybrid scheme already suggested in [5] seems to be a good supplement to the non-linear inversions, leading to a fine tuning of the inversion results. Linear inversion schemes are based on a linear relationship between variations of the recoverable parameters with respect to a reference environment and variations of observables between the actual signal and a simulated signal for the reference environment. Linear inversion schemes are usually applied in an iterative sense the convergence of which is based on the selection the reference environment which has to be as close as possible to the actual one.

In our approach, we will assume that modal arrivals are observed. The modal arrival times are defined on the basis of the group velocity for a specific mode n with associated eigenvalue vector $\mathbf{k_n} = [k_{n,0}, k_{n,1} \ldots, k_{n,J+1}]^T$, where J is the number of segments. Note that $k_{n,0}$ and $k_{n,J+1}$ correspond to the eigenvalues of the first and the last segments in which there is not any fluctuation of the sound speed profile.

Denoting by t_n^0 the arrival time of mode n for the reference environment, t_n the arrival time for the actual environment, assuming adiabatic propagation, and first order perturbations, we get the following formula associating variations of sound speed and modal travel times

$$\begin{aligned}\delta t_n &= t_n - t_n^0 = \\ &= \frac{\partial \delta k_n r}{\partial \omega} = \Bigg\{ -\int_0^r \frac{\omega^2}{k_n^0(r)} \int_0^{+\infty} \frac{1}{\rho(r,z)} \cdot \\ &\quad \frac{|u_n^0(r,z)|^2}{c_0^3(r,z)} \delta c(r,z) dz dr \Bigg\} \end{aligned} \tag{7}$$

where index 0 denotes quantities referred to the reference environment, n is the order of the corresponding normal mode and u_n^0 is the associated eigenfunction. The sound speed difference is defined as:

$$\delta c(z,r) = c(z,r) - c_0(z,r) \tag{8}$$

Discretizing the region supporting the eddy in J vertical segments of width $\Delta r_j,\ j = 1, \ldots, J$ as described in Figure 1 and having done a discretization in depth with I horizontal segments of width $\Delta z_i,\ i = 0, \ldots, I-1$ we get the relation:

$$\delta t_n = \partial_\omega \Bigg\{ -\omega^2 \sum_{j=1}^{J} \frac{\Delta r_j}{k_{nj}^0} \sum_{i=0}^{JI-1} \frac{\Delta z_i |u_{nij}^0|^2}{\rho_{ij} c_{0ij}^3} \delta c_{ij} \Bigg\} \tag{9}$$

as a discrete analogous of the formula (7). Here ∂_ω denotes partial derivative with respect to ω.

We further define the indices $q = q_{ij} = (j-1)I + i,\ i = 0, \ldots, I-1$ and $j = 1, \ldots, J$ and we write the previous relation in a version with two indices as:

$$\delta t_n = \partial_\omega \Bigg\{ -\omega^2 \sum_{q=0}^{JI-1} \frac{\Delta r_q}{k_{nq}^0} \frac{\Delta z_q |u_{nq}^0|^2}{\rho_{ij} c_{0q}^3} \delta c_q \Bigg\} \tag{10}$$

where $\Delta r_q = \Delta r_j,\ k_{nq}^0 = k_{ni}^0,\ u_{nq}^0 = u_{nij}^0,\ \Delta z_q = \Delta z_i,\ c_{0q} = c_{0ij},\ \rho_q = \rho_{ij}$ and $\delta c_q = \delta c_{ij}$.

Having expressed through relation (1) the sound speed variation in terms of EOFs, we can further simplify the system defined in (10) as following:

$$\begin{aligned}\delta c_q &= \sum_{\ell=0}^{MJ-1} \alpha_\ell f_{q\ell},\ \ell = Mj + k, \\ j &= 1, \ldots, J,\ k = 1, 2, ..M.\end{aligned} \tag{11}$$

By identification of N normal modes in the peaks of the waveforms of the reference signal we come to the following linear system:

$$\delta \mathbf{t} = \partial_\omega (\mathbf{GF}) \mathbf{a} \tag{12}$$

where $\delta \mathbf{t} \in \mathbf{R}^N$ is the vector with elements the N modal travel time differences between actual and estimated waveforms and $\mathbf{G} \in \mathbf{R}^{N \times JM}$, $\mathbf{F} \in \mathbb{R}^{JM \times MJ}$ are matrices which have elements described by:

$$\{\mathbf{G}\}_{nq} = -\omega^2 \frac{\Delta r_q \Delta z_q}{k_{nq}^0} \frac{|u_{nq}^0|^2}{\rho_q c_{0q}^3} \tag{13}$$

and

$$\{\mathbf{F}\}_{q\ell} = f_{q\ell}. \tag{14}$$

Denoting the quantity $\partial_\omega(\mathbf{GF})$ by $\mathbf{Q_0} \in \mathbb{R}^{N \times MJ}$ we get the kernel matrix of the system, which is written as

$$\delta \mathbf{t} = \mathbf{Q_0 a}, \tag{15}$$

where $\mathbf{Q_0} \in \mathbb{R}^{N \times MJ},\ \mathbf{a} \in \mathbb{R}^{MJ},\ \delta \mathbf{t} \in \mathbb{R}^N$

The system can be solved for the unknown EOF coefficients at the various layers by means of a *Singular Value Decomposition* (**SVD**) algorithm cropping the very small eigenvalues. Having obtained the EOF coefficients at the various segments, the sound speed profile is estimated using formula (2).

In our approach we will adopt an iterative scheme where the sound speed profile determined by (2) is treated as the reference environment for a new application of the whole linear scheme. In each step ν of the scheme, a new sound speed profile is estimated:

$$c_q^\nu(z) = c_q^{\nu-1}(z) + \delta c_q^\nu(z),\ \nu = 1, \ldots, L \tag{16}$$

Table I. Geoacoustic and Operational parameters

Reference Parameters	Actual Value
Water Depth (m)	400
Density of the Water (kg/m^3)	1000
Starting Range of the Eddy $R_1(m)$	3000
Ending Range of the Eddy $R_2(m)$	13000
Sound Velocity at the Bottom (m/s)	1600
Density of the Bottom (kg/m^3)	1500
Source Depth (m)	50
Receiver Depth (m)	50
Receiver Range (m)	20000
Central Frequency/Bandwidth (Hz)	50/20

Recall in our notation that 0 denotes quantities related to the reference environment and L is the maximum number of iterations defined on the basis of the convergence criterion :

$$\|\delta\mathbf{c}\| \leq \epsilon \quad (17)$$

where, $\|\delta\mathbf{c}\| = (\sum_{q=0}^{JM-1}(\delta c_q)^2)^{1/2}$ and ϵ is a predefined small number.

It should be pointed out that in order that the scheme is applicable, modal arrivals should be defined. The identification of the modal arrivals is a non-trivial task. The identification can be obtained automatically as suggested in [7], or manually by applying normal-mode analysis in the signal reproduced for the inversion results of the first phase (GA optimization) and excluding from the identification low energy peaks. It is not the purpose of this work however to focus on the modal identification scheme.

3. Inversion results

The hybrid scheme is applied in the environment presented in Table I and for the operational characteristics mentioned in the same table. The central frequency of the source has been considered low (may be non realistic for a typical tomography experiment) for reasons related to the speed of the inversion process which is very low when many propagating modes are to be calculated and coupled for a large number of high frequencies as this is necessary for the first stage of the inversion process.

The EOFs representing the eddy appear in Figure 3. We assume that $M = 3$ orders of EOFs adequately represent the expected variations of the sound speed profile. This is anyway a type of a-priori information exploited in our inversion scheme. In the case under consideration the reference sound speed profile is piecewise linear between the values $c_0(0) = 1500\ m/sec$, $c_0(100) = 1495\ m/sec$, and $c_0(400) = 1509\ m/sec$.

As described above, our inversion procedure consists of two phases. The first phase corresponds to the inversion via Statistical Characterization Scheme (**SCS**), in which there is no need for any physical

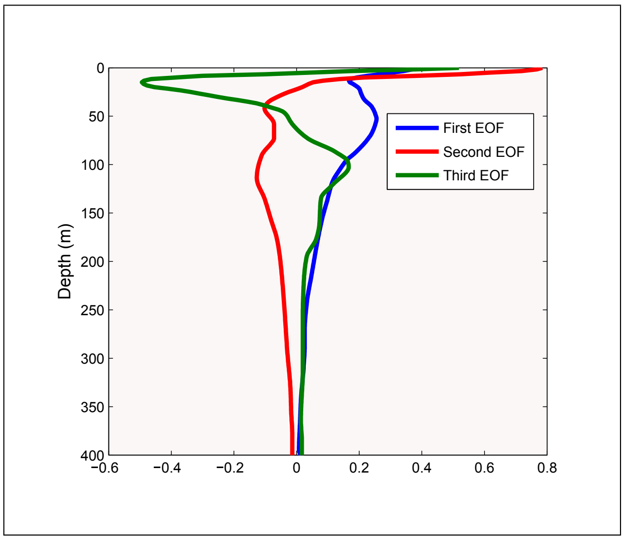

Figure 3. The three EOFs representing the sound speed anomaly for the test case.

Table II. The search space of the EOF coefficients

Order	Lower Bound	Upper Bound
1	-50	0
2	0	50
3	-30	0

observable identification and the second phase corresponds to the linear inversion scheme based on modal arrivals identification. Thus, we start from a search space for the coefficients of the EOFS describing the sound speed in the water column as defined in Table II (same bounds for all the segments) and the simulated signal presented in Figure 2. By means of the SCS assisted by the KLD Divergence and the GA after 50 Generations with crossover probability 0.8 and mutation 0.02 we come up with the a-posteriori statistical distributions of the final population (80 individuals) of the GA presented in Figure 4.

The differences between the best individual (denoted by cross) and the actual values for the EOF coefficients (denoted by star) are clearly seen there. Using these values, we simulate the signal in the time domain which is close to the actual one in terms of travel times of the individual modes as seen in Figure 5. We then proceed with the identification of the modal arrivals. Six (6) modal arrivals are identified and the differences between actual and estimated travel times (being the input to the linear inversion scheme of the second phase) are defined. The 6 modal arrivals correspond to modes (1st, 5th, 6th, 7th, 8th and 9th).

Starting an iterative process as described above, we come after just 4 iterations to convergence and obtain the results appearing in Table 2. Note that the system is under-determined (6 equations and 15 unknowns).

In Figure 6 the actual and recovered (SCS and LS) sound speed profiles are presented for each one of the 5 segments.

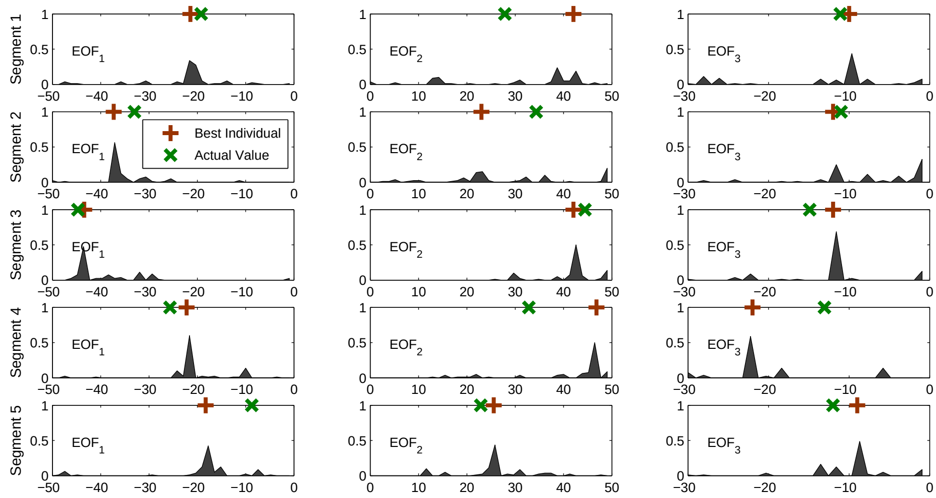

Figure 4. A-posteriori statistical distributions of the final GA population.

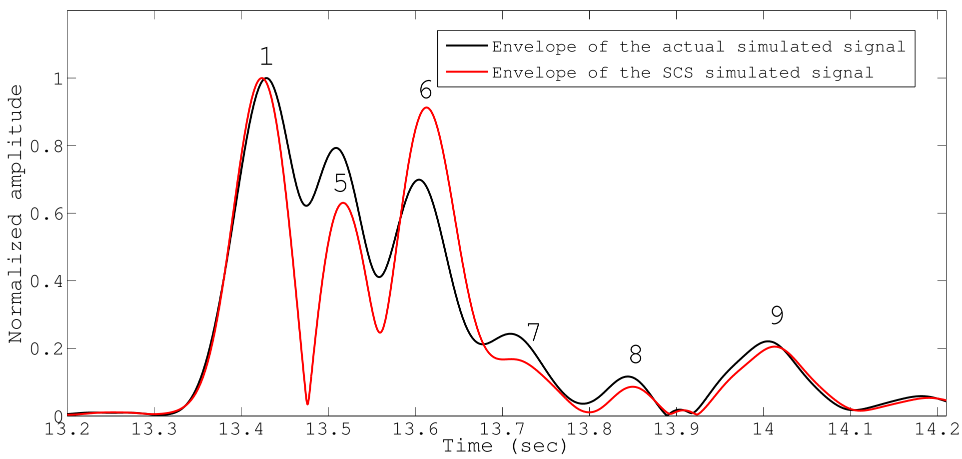

Figure 5. The envelope of the actual signal simulated using the inversion results from the GA.

Table III. Actual and recovered (SCS and LS) EOF coefficients for the test case inversion.

Segment	a_1 Actual↔SCS ↔LS	a_2 Actual ↔SCS↔ LS	a_3 Actual ↔ SCS ↔ LS
1	−19.21 ↔ −21.43 ↔ **−19.84**	27.85 ↔ 42.06 ↔ **35.76**	−11.10 ↔ −10.00 ↔ **−14.30**
2	−33.00 ↔ −37.30 ↔ **−36.91**	34.35 ↔ 23.02 ↔ **22.15**	−11.00 ↔ −12.00 ↔ **−15.50**
3	−44.71 ↔ −43.40 ↔ **−43.48**	44.44 ↔ 42.06 ↔ **41.26**	−14.89 ↔ −12.00 ↔ **−15.35**
4	−25.66 ↔ −22.22 ↔ **−20.29**	32.82 ↔ 46.83 ↔ **45.99**	−13.01 ↔ −22.00 ↔ **−26.84**
5	−8.72 ↔ −18.27 ↔ **−10.33**	22.88 ↔ 25.56 ↔ **24.61**	−12.01 ↔ −9.00 ↔ **−13.32**

We observe a slight improvement of the inversion results obtained by the non-linear inversion scheme, which are already considered as reliable. It should be noted however, that the hybrid scheme lacks consistency between the vertical segments, as the variation of the sound speed profile in each one of these is treated as independent with respect to the others, which is definitely not a physical condition.

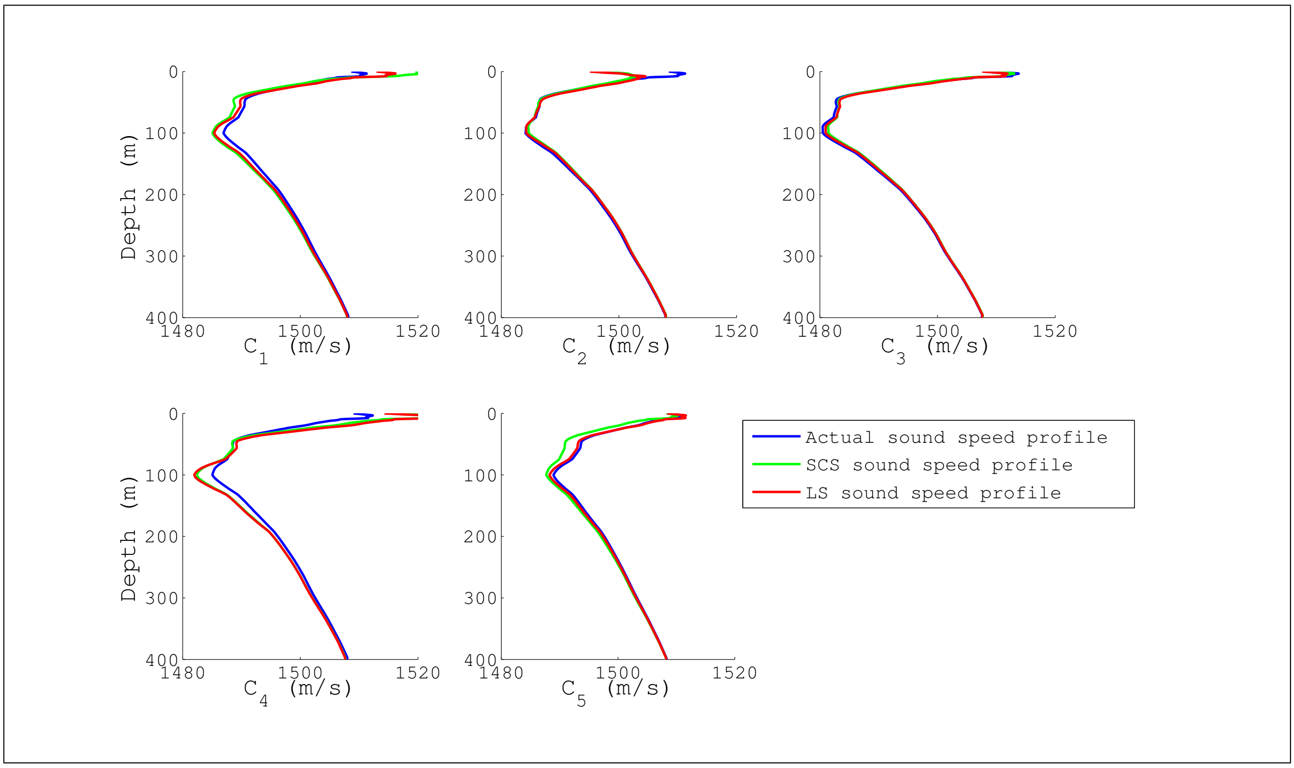

Figure 6. Actual and approximated sound speed profile in the water.

4. CONCLUSIONS

We presented here a two-phase hybrid inversion scheme for the estimation of the sound speed profile in the water column, based on the measurement of the acoustic field due to a known source at a single hydrophone. The second phase is based on a first order sensitivity kernel associating sound speed perturbations with modal travel time variations. The physical observables are the modal arrivals which should be identified in the recorded acoustic signal. A non-linear inversion scheme based on a statistical characterization of the acoustic signal and a Genetic Algorithm is used in the first phase. Using synthetic data in a shallow water environment it has been demonstrated that the suggested approach can be applied in range-dependent cases, with the example presented here corresponding to the recovery of the structure of a cold eddy. Of course, additional study is required to assess possible limitations of the hybrid approach in range-dependent environments especially in what concerns the sensitivity of the sensitivity kernel. Also the acceleration of the scheme especially in the first phase corresponding to the non-linear inversion is necessary for a practical application of the whole method.

5. ACKNOWLEDGMENTS

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