Inversion of acoustical data from the "Shallow Water 06" experiment by statistical signal characterization

Michael Taroudakis[a] **and Costas Smaragdakis**[a),b)]
Department of Mathematics and Applied Mathematics, University of Crete, University Campus, 70013, Heraklion, Crete, Greece
taroud@math.uoc.gr, kesmarag@tem.uoc.gr

N. Ross Chapman
School of Earth and Ocean Sciences, University of Victoria, P.O. Box 3055, Victoria, British Columbia, V8W3P6, Canada
chapman@uvic.ca

Abstract: This paper presents an application to validate an acoustic signal characterization scheme for ocean acoustic tomography and geoacoustic inversions proposed by Taroudakis, Tzagkarakis, and Tsakalides [J. Acoust. Soc. Am. **119**, 1396–1405 (2006)] using data from an experiment at sea. The data were collected during the Shallow water '06 (SW06) experiment off the New Jersey Continental Shelf and the inversion results (sea-bed geoacoustic parameters and source range) are compared with those reported from the same data by Bonnel and Chapman [J. Acoust. Soc. Am. **130**(2), EL101–EL107 (2011)]. The comparison and the signal reconstruction using estimated values of the model parameters are satisfactory indicating that the new signal characterization method is useful for practical applications of acoustical oceanography.

PACS numbers: 43.30.Pc [JL]
Date Received: May 21, 2014 **Date Accepted:** September 7, 2014

1. Introduction

In 2006, a multipurpose experiment Shallow Water '06 (SW06) was carried out off the coast of New Jersey.[1] For geoacoustic inversion purposes, light bulbs were deployed in the water, and their implosions were recorded at a distance about 7 km away from the source location. The geoacoustic model is a shallow water waveguide with range-independent characteristics. The water depth (79.1 km) was known. The sound speed profile shown in Table 1 can be considered as an average sound speed profile for this region. The sea-bed is described as a two layer medium with a homogeneous semi-consolidated layer of approximately 20 m thickness overlying a harder substrate characterized by an interface known as the R-reflector.[2] For the purposes of our study, both layers will be considered as fluids. The recordings were made at an array of hydrophones but in the present work a single recording at the depth of 67.1 m was used. One of the tasks of the experiment was the validation of geoacoustic inversion schemes. To this end, the sound speeds and densities of the sediment layer and the substrate as well as the thickness of the layer and the actual range of the source were the unknowns to be recovered. Bonnel *et al.*[3,4] presented inversion results based on dispersion curves estimation using warping operators to improve mode separability. It was our purpose to use the same data from the experiment to test the applicability of a

[a)]Also at FORTH, Institute of Applied and Computational Mathematics, Heraklion, Crete, Greece.
[b)]Author to whom correspondence should be addressed.

Table 1. The sound speed profile in the water column.

Depth z (m)	Sound Speed Profile c_w (m/s)
0.0	1525.0
10.0	1525.0
27.0	1483.0
79.1	1490.0

new method of signal characterization based on the statistical distributions of the wavelet sub-band coefficients already presented in Ref. 5 and validated with simulated data as in Refs. 6–8, in real world applications. This statistical characterization scheme will be denoted as "SCS." An important issue in this respect is the modeling of the source excitation function which will be briefly presented in the next section. Section 3 presents the inversion results to be discussed in Sec. 4.

2. Simulation of the source excitation function

For inversion purposes, the SCS is associated with an optimization process based on repeated simulations of the received signal for a class of candidate environments. The simulations are made by calculating the system transfer function $H(\mathbf{x_s}, \mathbf{x_r}; \omega)$ at a specific angular frequency ω $(\omega = 2\pi f)$, where $\mathbf{x_s}$ and $\mathbf{x_r}$ are the source and receiver position vector, respectively. The pressure is obtained by multiplying the transfer function with the source excitation function $S(\omega)$,

$$p(\mathbf{x_s}, \mathbf{x_r}; \omega) = H(\mathbf{x_s}, \mathbf{x_r}; \omega)S(\omega). \tag{1}$$

Repeating this process for all frequencies in the effective signal bandwidth, we take the signal in the time domain by Inverse Fourier Transform. Here, the system transfer function is calculated using a forward propagation model, which in our case is the MODE1 program developed at FORTH and based on normal-mode representation of the acoustic field. Therefore, the source excitation function must be given or calculated prior to the application of the inversion algorithm. Raw data of the light bulb implosions implied that the effective bandwidth of the acoustic signals was from 30 to 200 Hz.[3] Thus, we filtered the raw data with a band-pass filter allowing frequencies in this band.

In the SW06 experiment, the light bulb implosion was set for a depth of 22 m using a messenger weight to break the bulb. Figure 1(a) presents the waveform of the

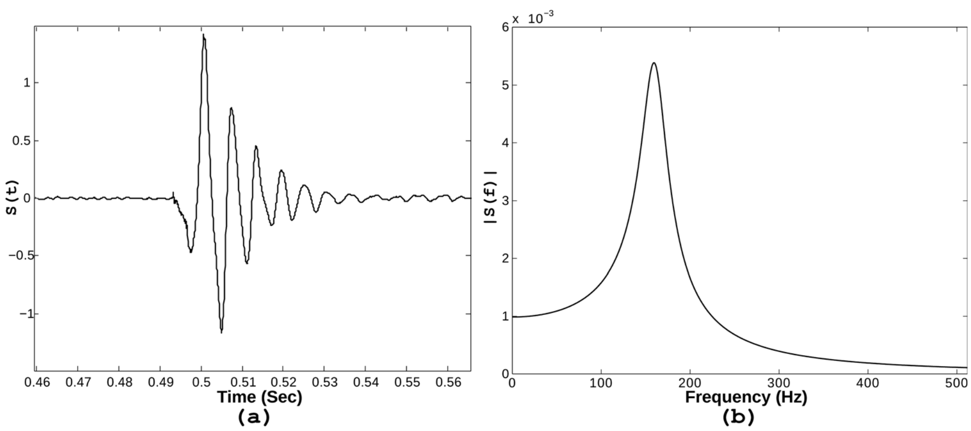

Fig. 1. (a) The actual recorded signal closed to the light bulb source. (b) The simulated signal in the frequency domain.

light bulb implosion recorded close to the source location. The implosion of the light bulb generates a signal in the water which consists of two parts.

The first part is characterized by a negative pressure phase because of a decrease in pressure during the collapse of the light bulb surface. This effect is a consequence of the instantaneous water flow into the light bulb volume.

The second and the most significant part is a damped sine wave as a result of the oscillation of the light bulb gas volume in the water. After the implosion, the gas forms a spherical void in the water with an initial radius. The void collapses due to the hydrostatic pressure to a minimum value and when the internal pressure overtakes the external one, the void starts to increase its volume. This process continues and a damped sine waveform is generated and transmitted in the water.

A damped sine waveform has the analytic form

$$S(t) = AH(t)\exp(-kt)\sin(2\pi f_c t), \quad (2)$$

where A is a normalization constant, H is the Heaviside function, f_c is the frequency of the pulse, and k is the decay parameter.

All these parameters should be estimated in order to model the actual excitation function of the experimental light bulb source.

The f_c can be estimated by measuring the time difference between two peaks of consecutive positive pulses and then by taking its inverse. For the signal illustrated in Fig. 1(a), this value is equal to 160 Hz.

The decay parameter is estimated by measuring the amplitude of two consequent peaks $S(t_1)$ and $S(t_2)$ of the signal given the times corresponding to these peaks. In the example, if we know the pairs of coordinates $[t_1, S(t_1)]$ and $[t_2, S(t_2)]$ then the k parameter has the value

$$k = \frac{1}{t_2 - t_1}\ln\frac{S(t_1)}{S(t_2)}. \quad (3)$$

For our data, the proper value of k is equal to 92.8. Note that the estimation of the decay parameter can be found for any pair of consecutive peaks without any significant difference.

It is interesting to notice that the Fourier transform of the modeled source has the analytic form

$$S(\omega) = A\frac{\omega_c}{(k + i\omega)^2 + \omega_c^2}, \quad (4)$$

where ω denotes the angular frequency. Figure 1(b) illustrates the marginal spectrum of the source excitation function for frequencies in the range [0, 512] Hz.

3. The inversion procedure and results

Following the work by Taroudakis *et al.*,[5] an acoustic signal is characterized by the statistical parameters of the wavelet sub-band coefficients. In our case, we perform a multiresolution analysis employing the 1D wavelet transform as in Ref. 9. For typical signals used in applications of Acoustical Oceanography, it has been shown that the wavelet coefficients obey a symmetric alpha stable distribution (SaS) characterized by two parameters (α, γ).[10] Daubechies 4 (db4) wavelet has been used in our analysis following previous works on statistical acoustical signal characterization.[1]

For a L-level wavelet analysis, the signal can be characterized by L detailed and 1 approximation coefficient vectors Φ, each one of which consisting of only two elements. Hence, the signal feature is represented by a vector **d** as follows:

$$S \leftrightarrow \{\Phi^0, \ldots\ldots, \Phi^L\} \leftrightarrow \mathbf{d} = [(\alpha^0, \gamma^0, \alpha^1, \gamma^1, \ldots, \alpha^L, \gamma^L)]^T, \quad (5)$$

where T denotes the transpose.

It has been shown in previous works[6–8] that $L = 3$ is an adequate limit of the multilevel analysis of typical underwater acoustic signals. Therefore in our work, we have used a feature vector in $\mathbb{R}$ (Ref. 8),

$$\mathbf{d} = [\alpha^0, \gamma^0, \alpha^1, \gamma^1, \alpha^2, \gamma^2, \alpha^3, \gamma^3]^T \tag{6}$$

for signal characterization.

For the geoacoustic inversion experiment under consideration, the six unknown parameters can be described by a vector in $\mathbb{R}$ (Ref. 6) as

$$\mathbf{m} = [r, c_p, \rho_p, h, c_b, \rho_b]^T. \tag{7}$$

Using the concepts described above, the following non-linear inverse problem is formulated: Given a single acoustic signal characterized by the vector **d**, estimate the model parameters **m** given a certain propagation model and the specific signal characterization scheme (SCS), jointly described by means of the vector function **T** through an equation of the form

$$\mathbf{T}(\mathbf{d}, \mathbf{m}) = 0. \tag{8}$$

The problem being non-linear and ill-posed is amenable to a solution based on an optimization process, where the cost function is chosen so that the statistical character of the feature vector is exploited. It has been shown that the Kullback-Leibler Divergence (KLD)[11] written analytically for the case of SaS distributions by the following closed form relation:

$$D_s(S_1, S_2) = \sum_{k=0}^{L} \left\{ \ln\left(\frac{c_2^k}{c_1^k}\right) - \frac{1}{\alpha_1^k} + \left(\frac{\gamma_2^k}{\gamma_1^k}\right)^{\alpha_2^k} \frac{\Gamma\left[\left(\alpha_2^k + 1\right)/\alpha_1^k\right]}{\Gamma\left(1/\alpha_1^k\right)} \right\}, \tag{9}$$

where $\Gamma(x)$ is the Gamma function and

$$c_i^k = \frac{2\Gamma\left(1/\alpha_i^k\right)}{\alpha_i^k \gamma_i^k}, \quad i = 1, 2, \quad k = 0, \ldots, L, \tag{10}$$

is an appropriate cost function[7] expressing the difference between signals S_1 and S_2.

The optimization process in our work is controlled by a Genetic Algorithm (GA) (see Refs. 7 and 8). Here, the GA was applied for 100 generations of 80 individuals each, with probabilities of crossover 0.8 and mutation 0.02. The search space was chosen to be exactly the same as in Ref 3.

For each set of the unknown parameters taken from the search space, the signal is simulated in the time domain followed by its statistical characterization, and by using the KLD, the signal is compared with the received one.

Figure 2 presents the *a posteriori* probability distribution of the individual members of the final population indicating by cross symbol the best individual of the GA algorithm and by "×" the inversion results obtained by Bonnel and Chapman. As we consider the best individual as the most probable solution to the inverse problem, these values should be compared with Bonnel and Chapman inversion results. Both inversion results are presented in Table 2. The results by Bonnel and Chapman[2] are denoted as "B-C" and the results by the SCS are denoted as "SCS."

The comparison of the results obtained by the two totally different inversion schemes can be summarized as following:

The source range estimated by both methods is practically the same. The sound speed in the sediment layer is estimated by the two methods with a difference of approximately 13 m/sec. The difference for most applications of acoustical oceanography is not considered important. The sound speed in the substrate is estimated by the two methods with a difference of nearly 260 m/sec. Although this difference in absolute

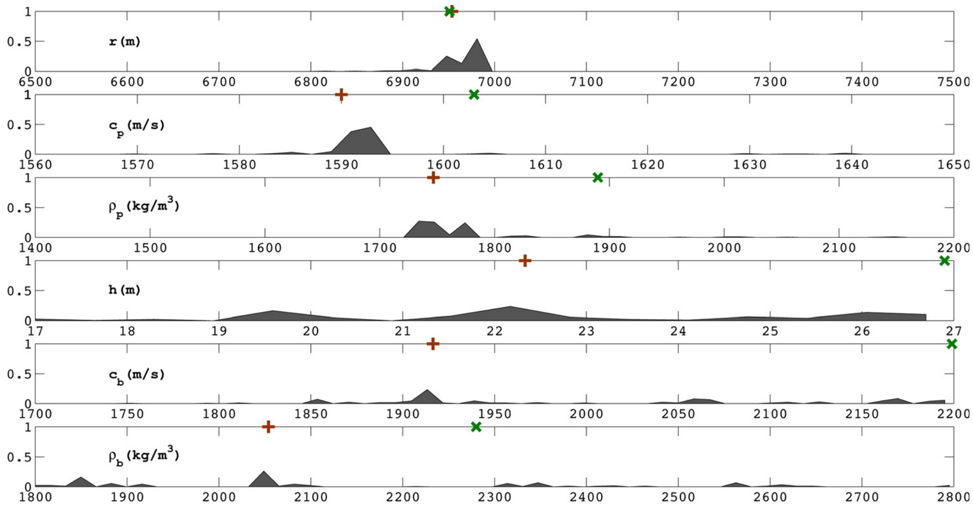

Fig. 2. (Color online) *A posteriori* statistical distributions of the final population of the GA. A cross denotes the model parameter corresponding to the best individual of the final population according to SCS and × denotes the value of the parameter estimated by Bonnel and Chapman.

terms seems considerable, both values indicate a higher sound speed as expected for the more consolidated sediment material below the R-reflector. Moreover, the light bulb signal does not penetrate with great strength to the deeper depths in the sediment, so the data do not contain much information about the substrate. The densities of the sediment layer and the substrate are estimated by the two methods with values exhibiting considerable differences. It is well known, however, that the density is among the parameters that are generally not well estimated by acoustical means with high accuracy. Finally, the sediment thickness estimates from the two methods were reasonably close and consistent with the values expected for the depth of the R-reflector at the base of the layer.[2–4]

By comparing the signal waveform simulated using the inversion results of the two methods and the MODE1 program for the calculation of the system transfer function (Fig. 3), it can be seen that the differences are not significantly large, especially for the initial times that are characteristic of the low order modes. This is an interesting result, suggesting that the inversion scheme based on the statistical characterization of the recorded signal can give estimations of the model parameters that reproduce the signal in similar quality with respect to other inversion schemes. Moreover, the two sets of model parameters lead to simulated signals with very similar shape with the actual signal especially in the early part. This is an additional indication of the reliability of the inversion results.

Table 2. Inversion results.

Unknown Parameters	Unit	B-C estimations	SCS estimations
Range	r (m)	6951.0	6953.7
Sediment sound speed	c_p (m/s)	1603.0	1590.0
Sediment density	ρ_p (g/m^3)	1890.0	1746.8
Sediment thickness	h (m)	26.9	22.3
Substrate sound speed	c_b (m/s)	2199.0	1916.4
Substrate density	ρ_b (g/m^3)	2280.0	2054.0

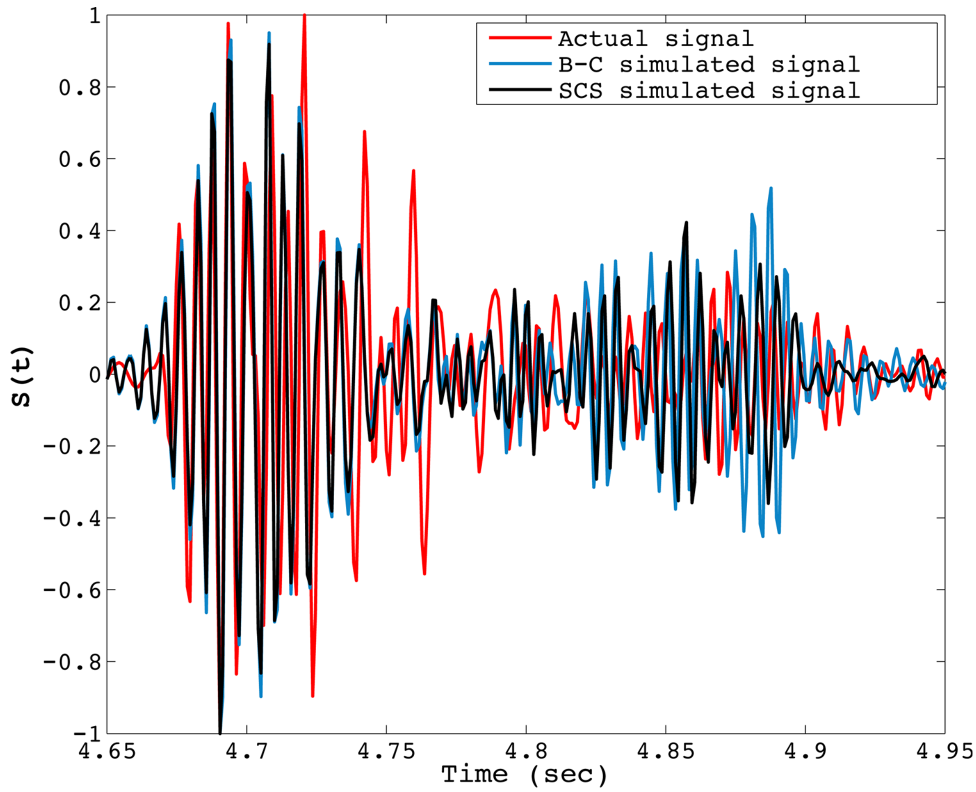

Fig. 3. (Color online) Actual and Simulated signals using model parameters estimated by the two inversion methods.

4. Conclusions

The paper presented a first attempt to apply a new method of acoustic signal characterization for geoacoustic inversions with real data. A Genetic Algorithm was used in the optimization process associated with the characterization scheme and the source excitation function was modeled by a Gaussian function. The inversion results were compared with those reported by Bonnel and Chapman,[3] and it was shown that they lead to a similar reconstruction of the acoustic signal. The obvious differences between the actually recorded signal and the simulated ones can be attributed to the modeling of the source and the presence of noise which was not taken into account in the simulated signals. The important conclusion from the work presented here is that the acoustic signal characterization scheme based on the statistics of the sub-band wavelet coefficients validated so far by simulations only can indeed be used for geoacoustic inversions in real world experiments.

Acknowledgments

We would like to express our thanks to David Knobles and Preston Wilson for their valuable help in providing the light bulb source waveforms. This work was performed in the framework of the PEFYKA project within the KRIPIS Action of the GSRT. The project is funded by Greece and the European Regional Development Fund of the European Union under the NSRF and the O.P. Competitiveness and Entrepreneurship. N.R.C. acknowledges support from Office of Naval Research Ocean Acoustics Code 321.

References and links

[1]D. J. Tang, J. Moum, J. Lynch, P. Abbot, R. Chapman, P. Dahl, T. Duda, G. Gawarkiewicz, S. Glenn, J. Goff, H. Graber, J. Kemp, A. Maffei, J. Nash, and A. Newhall, "Shallow Water' 06—A joint acoustic propagation/nonlinear internal wave physics experiment," Oceanography (Wash. D.C.) **20**, 156–167 (2007).

[2]J. A. Goff, B. J. Kraft, L. A. Mayer, S. G. Schock, C. K. Sommerfield, H. C. Olson, S. P. S. Gulick, and S. Nordfjord, "Seabed characterization on the New Jersey middle and outer shelf: Correlatability and spatial variability of seafloor sediment properties," Mar. Geol. **209**,147–172 (2004).

[3]J. Bonnel and N. R. Chapman, "Geoacoustic inversion in a dispersive waveguide using warping operators," J. Acoust. Soc. Am. **130**(2), EL101–EL107 (2011).

[4]J. Bonnel, S. E. Dosso, and N. R. Chapman, "Bayesian geoacoustic inversion of single hydrophone light bulb data using warping dispersion analysis," J. Acoust. Soc. Am. **134**(1), 120–130 (2013).

[5]M. Taroudakis, G. Tzagkarakis, and P. Tsakalides, "Classification of shallow water acoustic signals via alpha stable modeling of the one dimensional wavelet coefficients," J. Acoust. Soc. Am. **119**, 1396–1405 (2006).

[6]G. Tzagkarakis, M. I. Taroudakis, and P. Tsakalides, "A statistical geoacoustic inversion scheme based on a modified radial basis functions neural network," J. Acoust. Soc. Am. **122**, 1959–1968 (2007).

[7]M. I. Taroudakis and C. Smaragdakis, "On the use of Genetic Algorithms and a statistical characterization of the acoustic signal for tomographic and bottom geoacoustic inversions," Acta Acust. Acust. **95**, 814–822 (2009).

[8]M. Taroudakis and C. Smaragdakis, "Inversions of statistical parameters of an acoustic signal in range-dependent environments with applications in ocean acoustic tomography," J. Acoust. Soc. Am. **134**, 2814–2823 (2013).

[9]S. Mallat, "A theory for multiresolution signal decomposition: The wavelet representation," IEEE Trans. Pattern Anal. Mach. Intell. **11**, 674–692 (1989).

[10]J. P. Nolan, "Parameterizations and modes of stable distributions," Stat. Probab. Lett. **38**, 187–195 (1998).

[11]S. Kullback, *Information Theory and Statistics* (Dover, New York, 1998).